Color Symmetry in Plane Patterns

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The symmetries of a pattern form a *group*

A symmetry is a transformation that leaves the pattern invariant

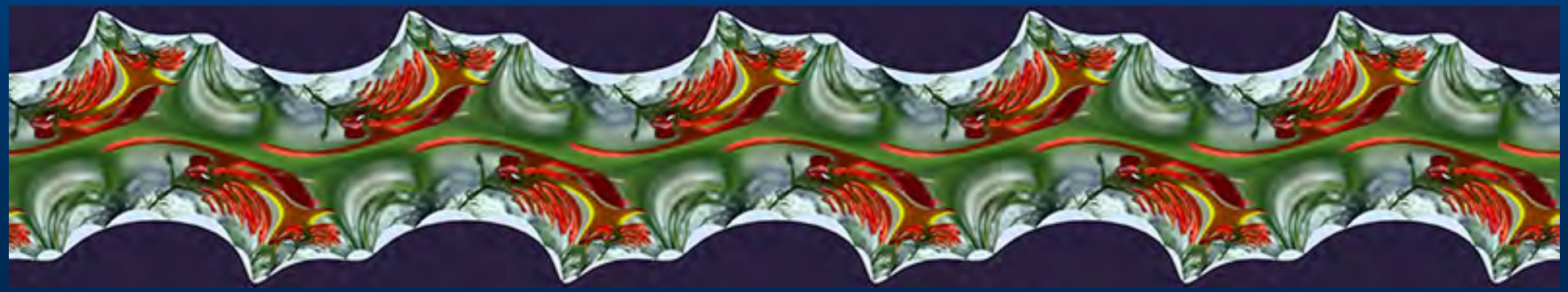
Rosettes, friezes, wallpaper

My origin story: grating def'ns of "pattern"

"A frieze pattern is a set of points that is invariant under..."

...FƎFƎFƎFƎ...

Set of points? A pattern is a function!

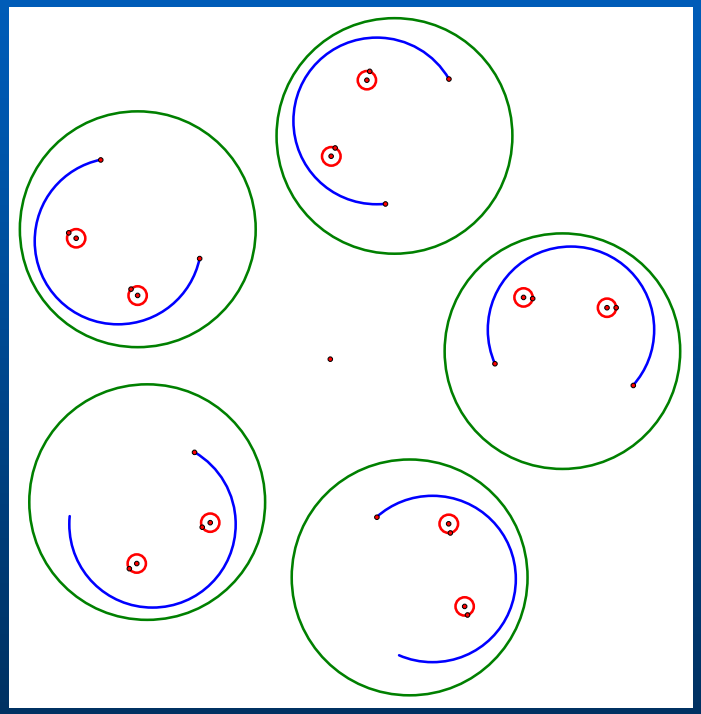


Stumbled on color symmetry by accident

Rendered in Excel!

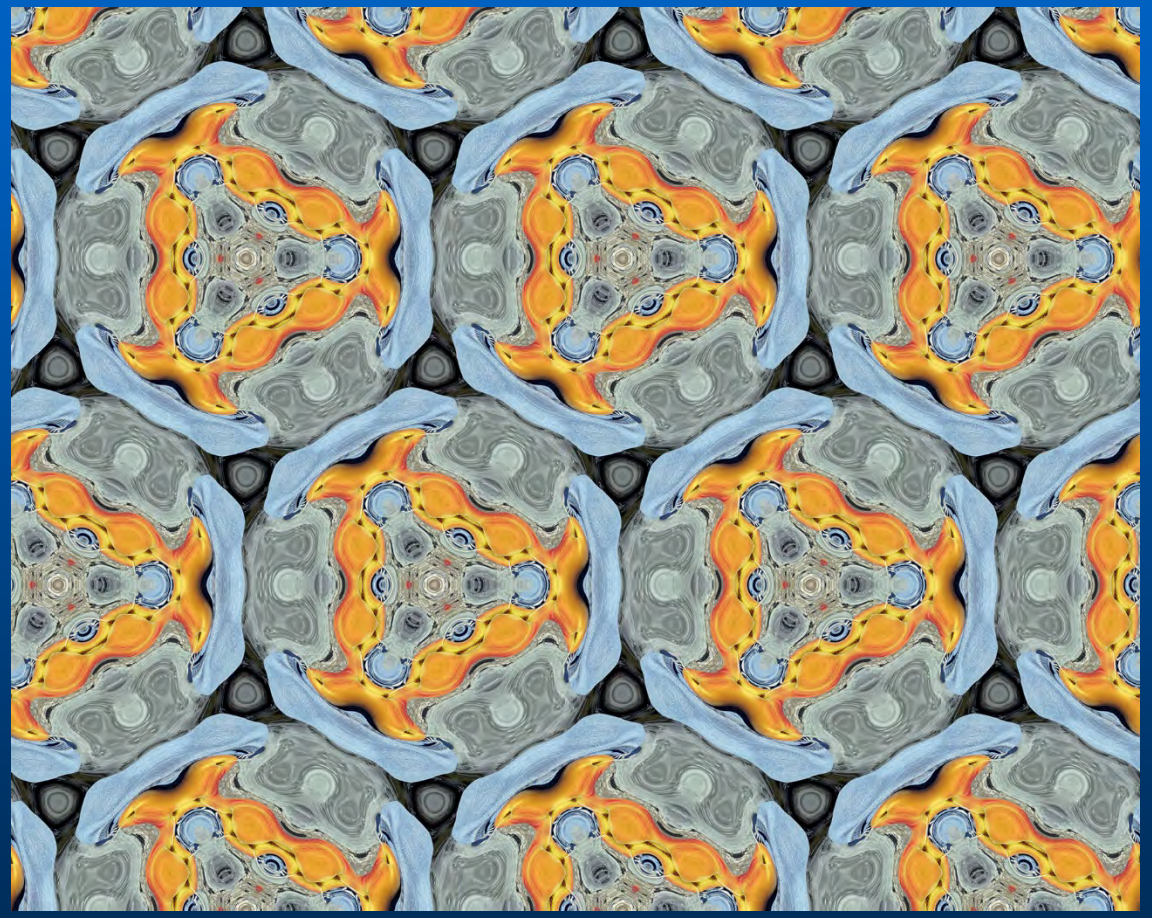
My origin story: grating def'ns of "pattern"

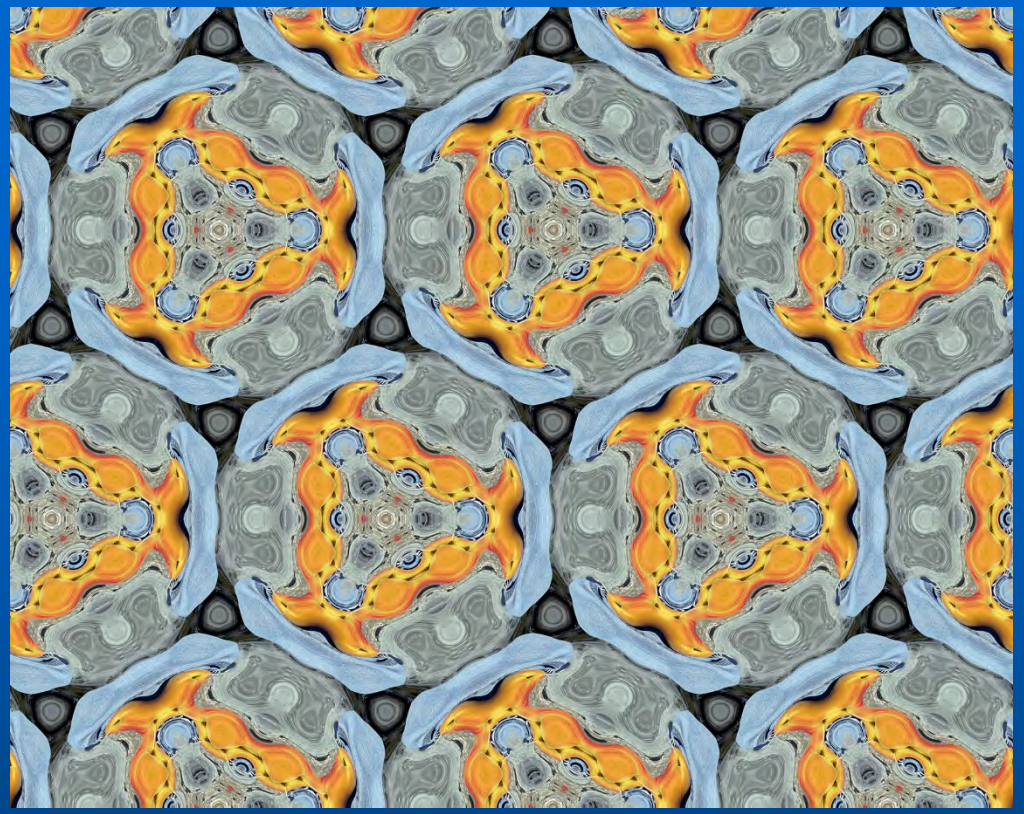
"A pattern is obtained by repeating a *motif*..."

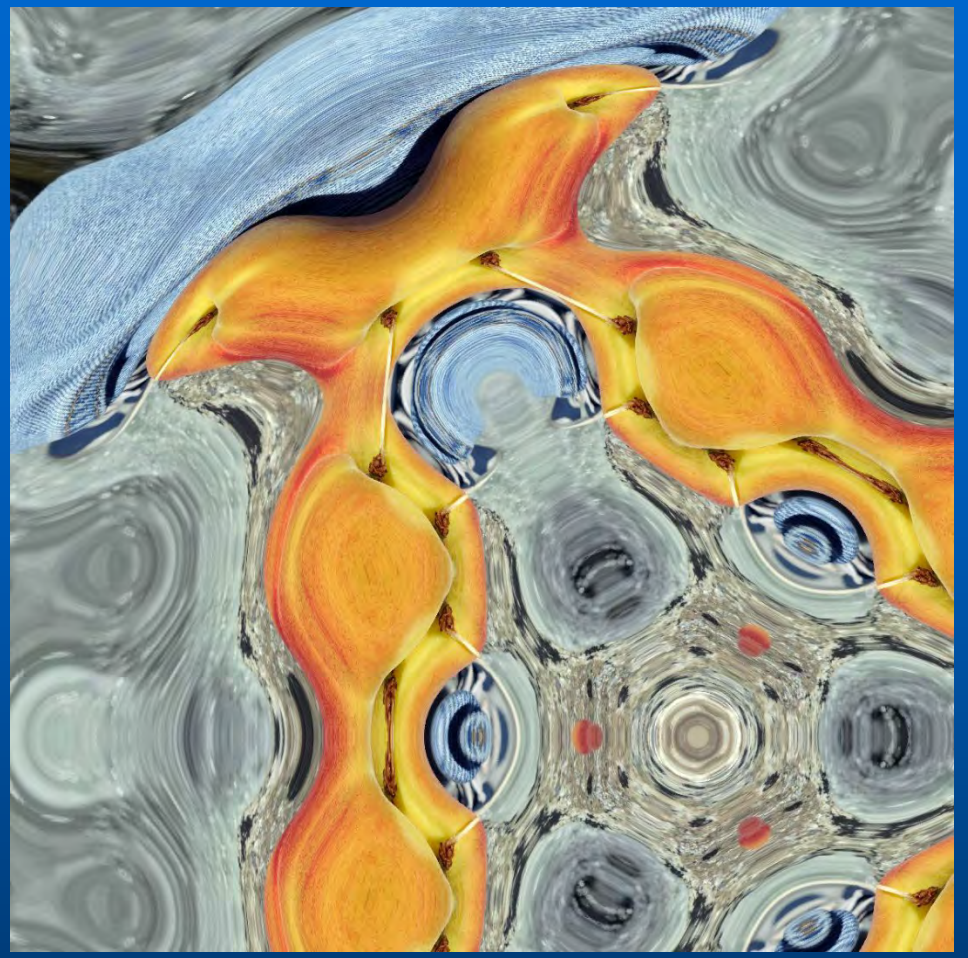


My response: Patterns are made from waves!

Waves

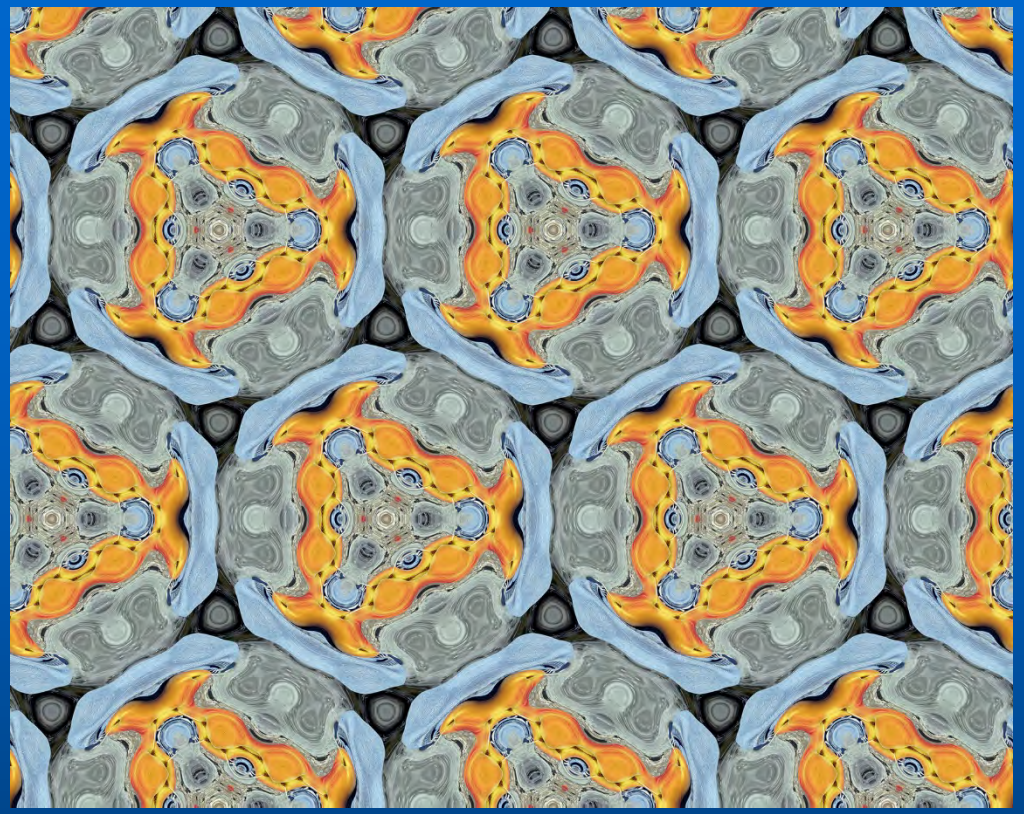


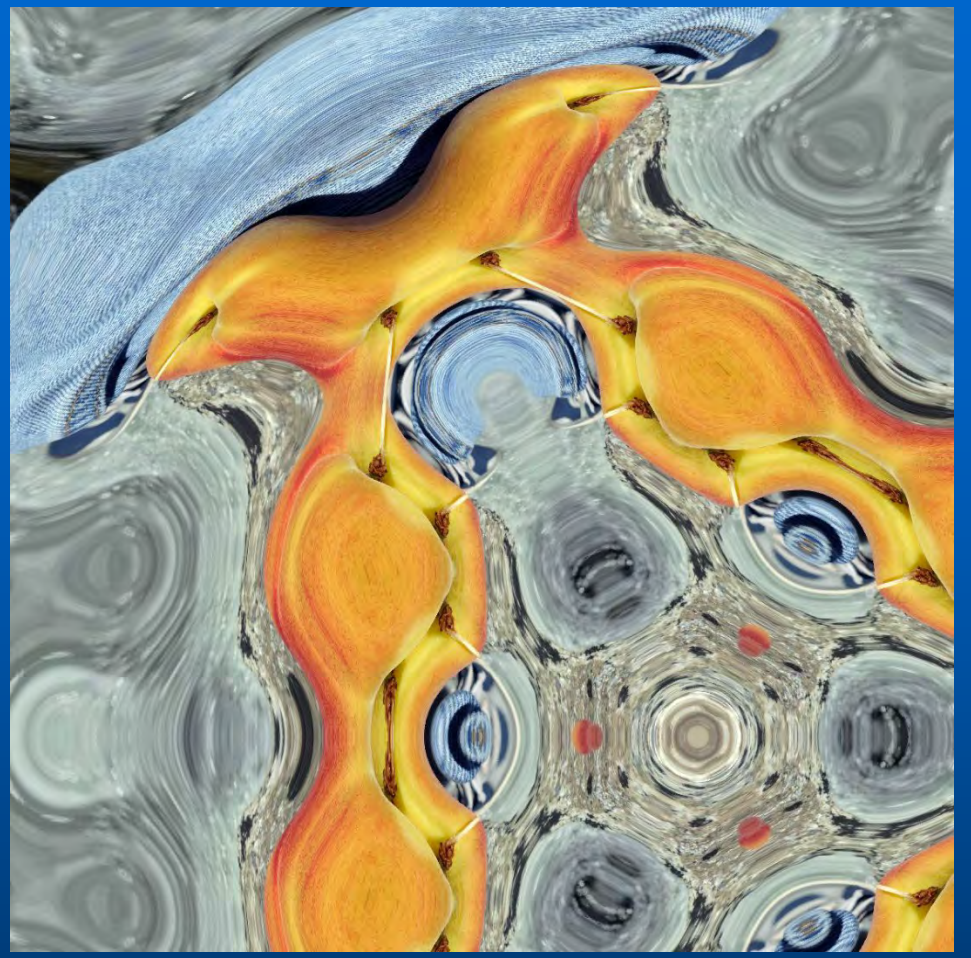




SymmetryWorks software allows you to play too.

Public domain software written by students at Bowdoin College (and SCU)

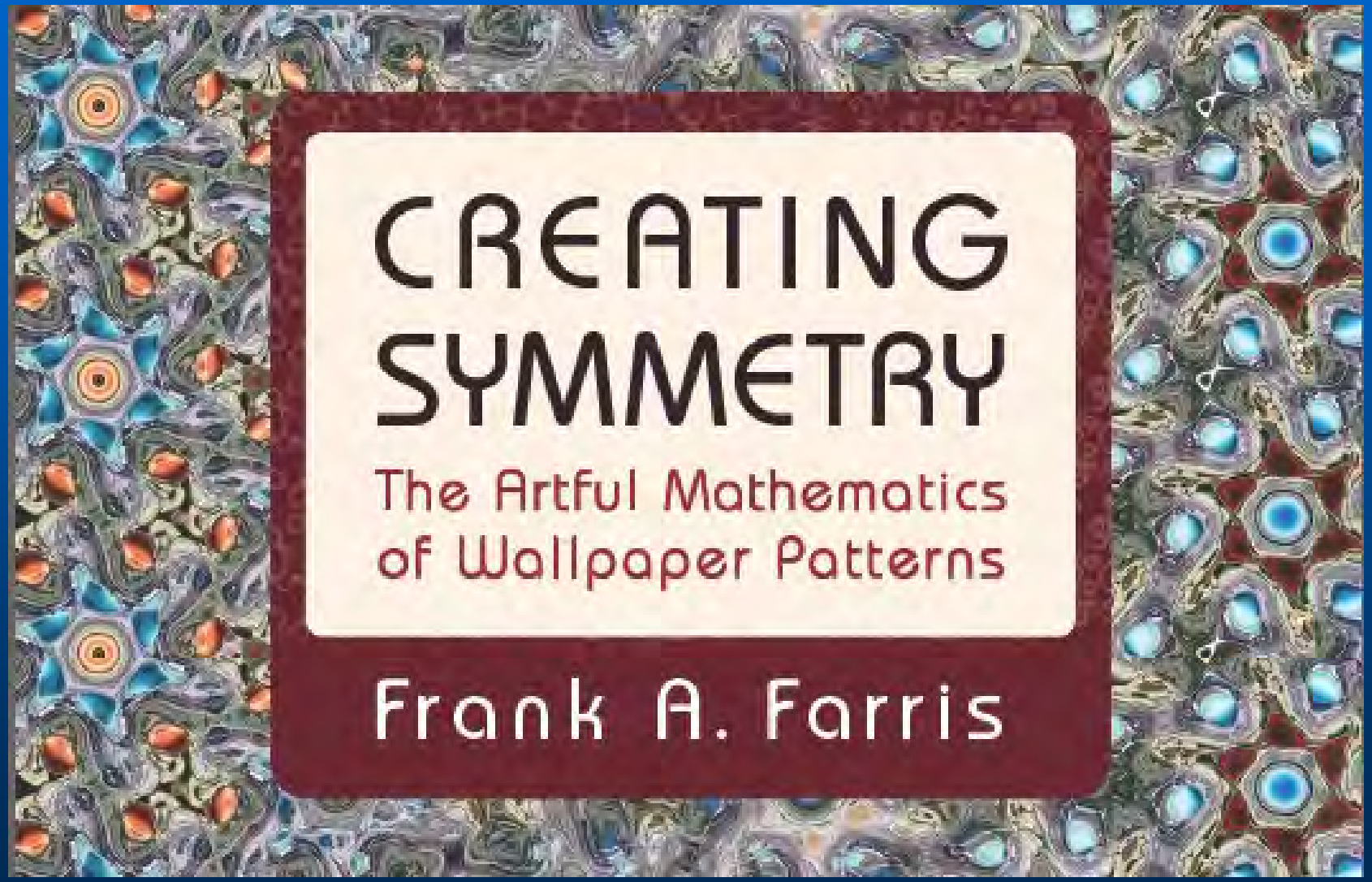


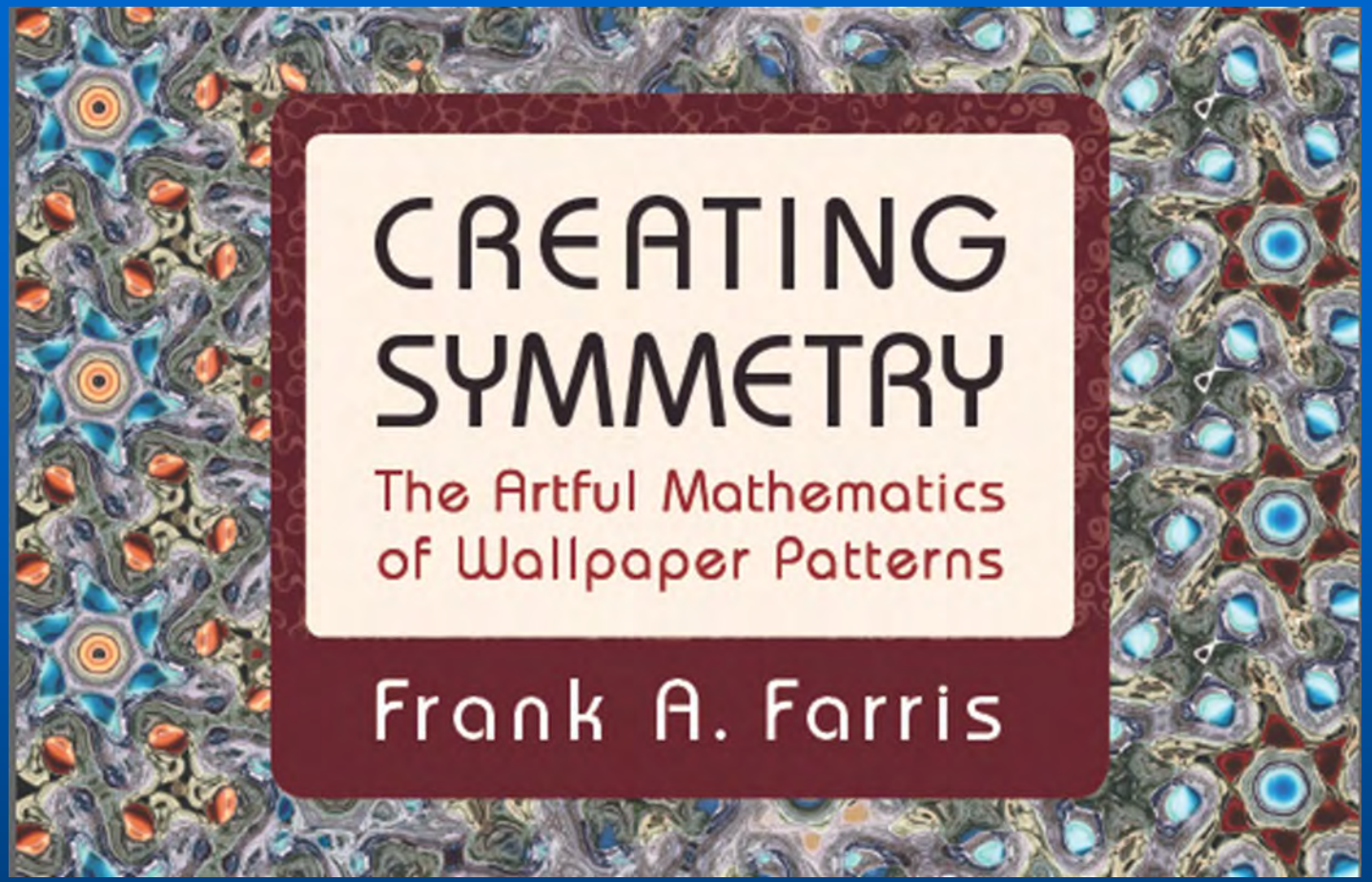


$$f(g(z)) = f(z) \text{ for } g \in G$$

Connect function-theoretic approach to color symmetry

Read the details:



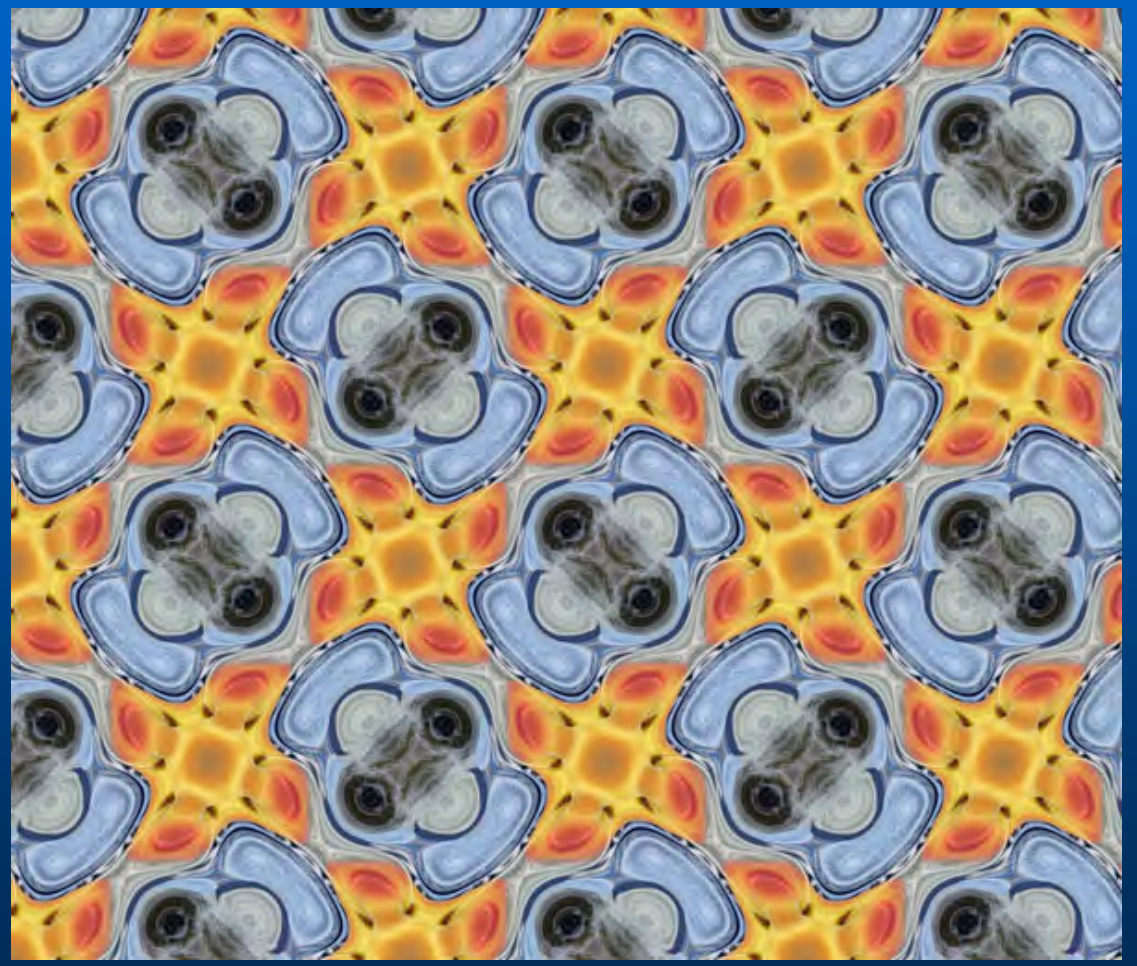


- Classifying patterns
- Two-color symmetry
- Three-color symmetry

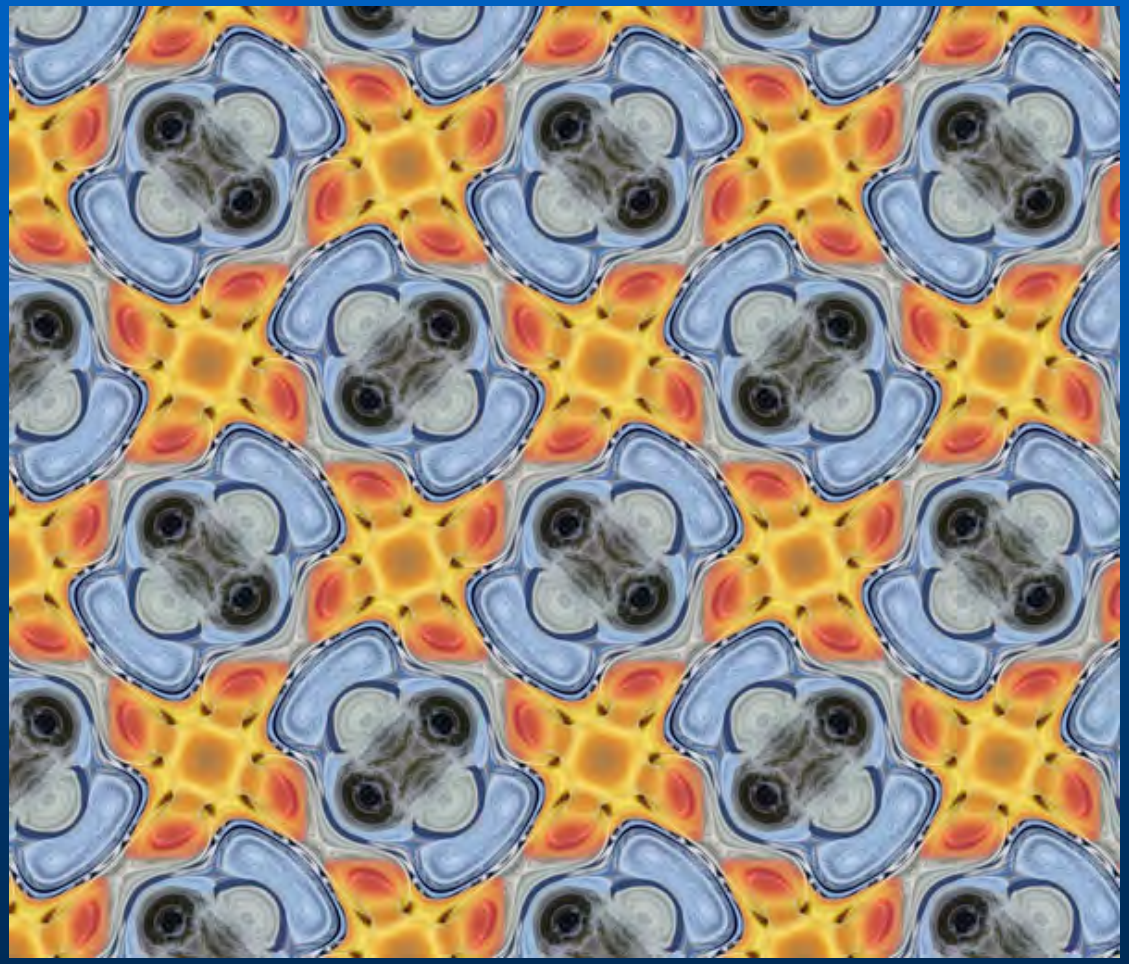
Connect to textiles in the real world?

Classifying patterns

Wallpaper: Translational symmetry along two independent axes

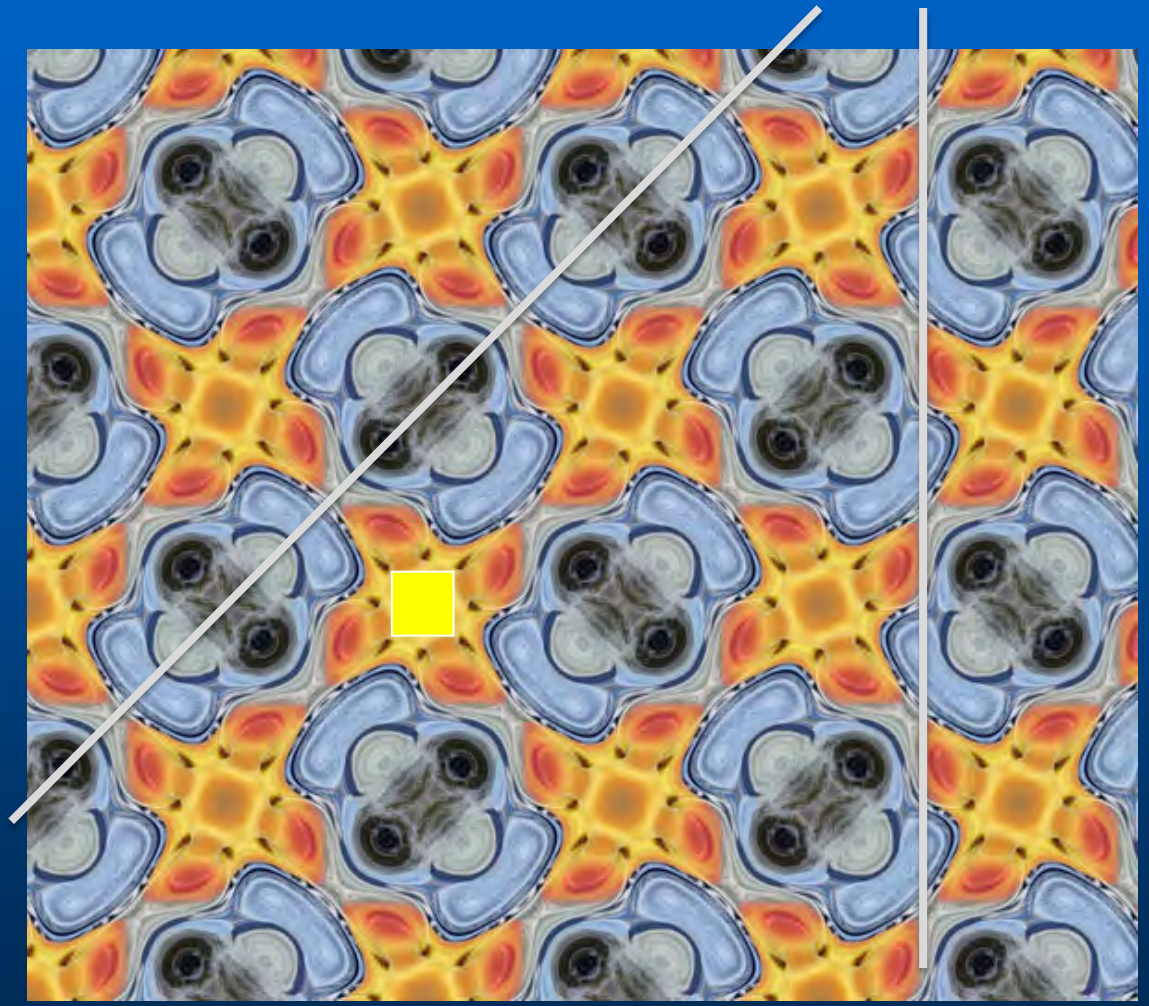


Classify patterns by symmetries



Too many to list!

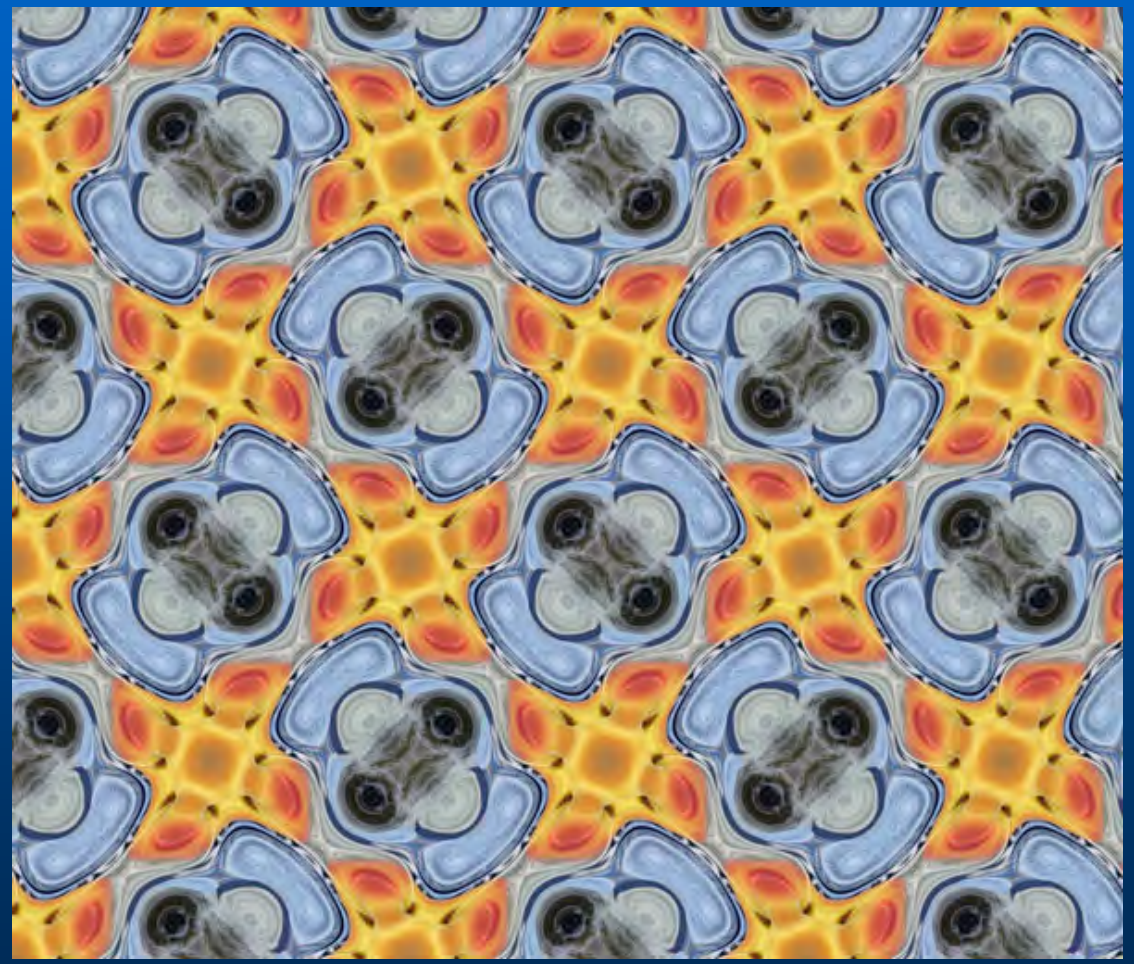
Mirrors



4-Centers

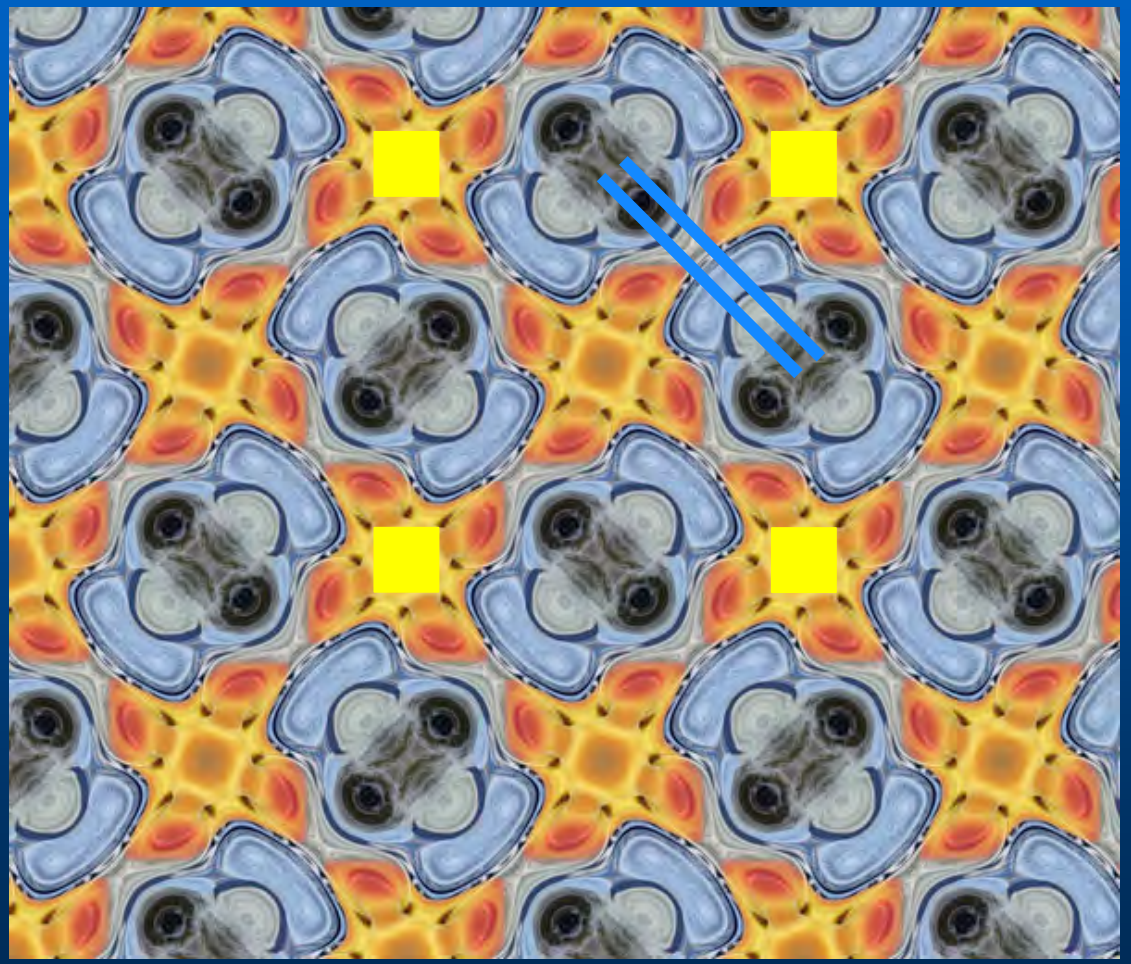
Glide reflections

Group concept: Compose two symmetries, get a symmetry

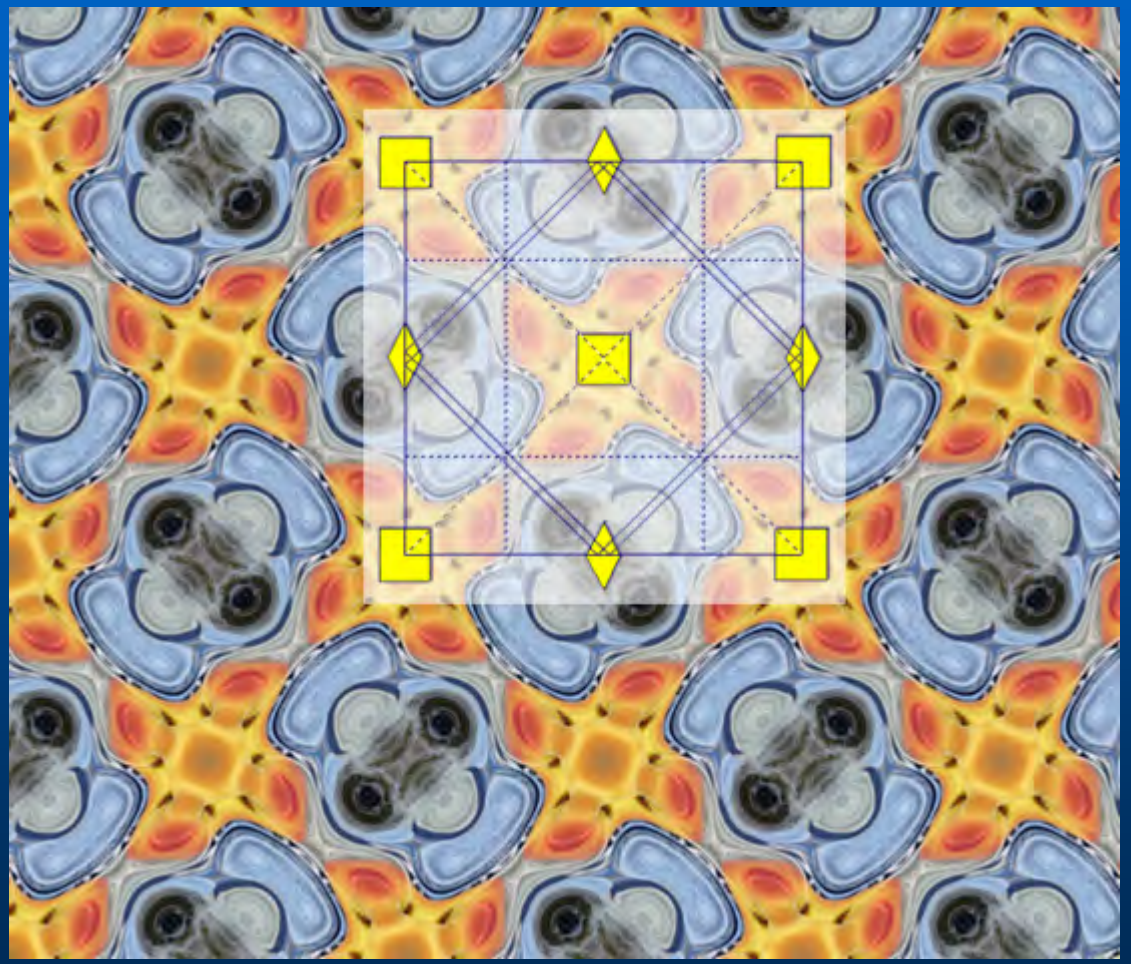


"freedom and constraint"

Learn how to draw a “fundamental cell”

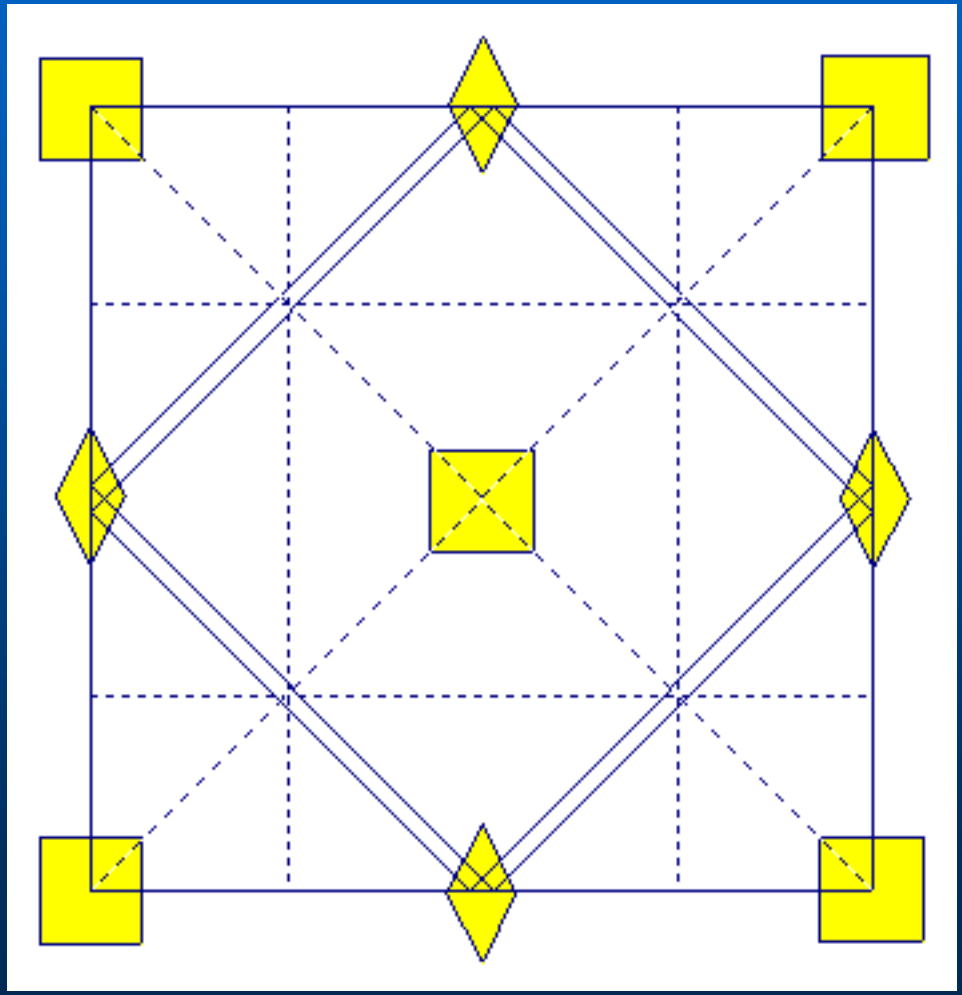


Complete cell diagram with…

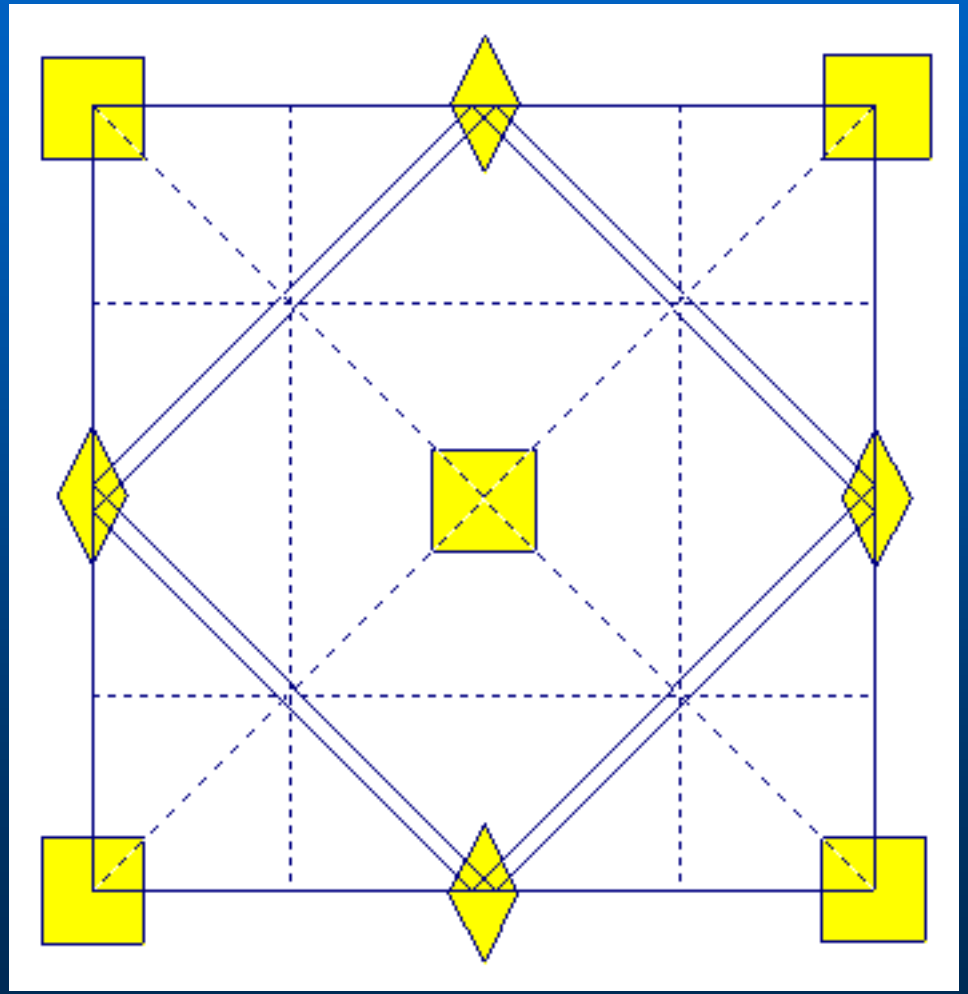


Group concept: a few symmetries can generate all

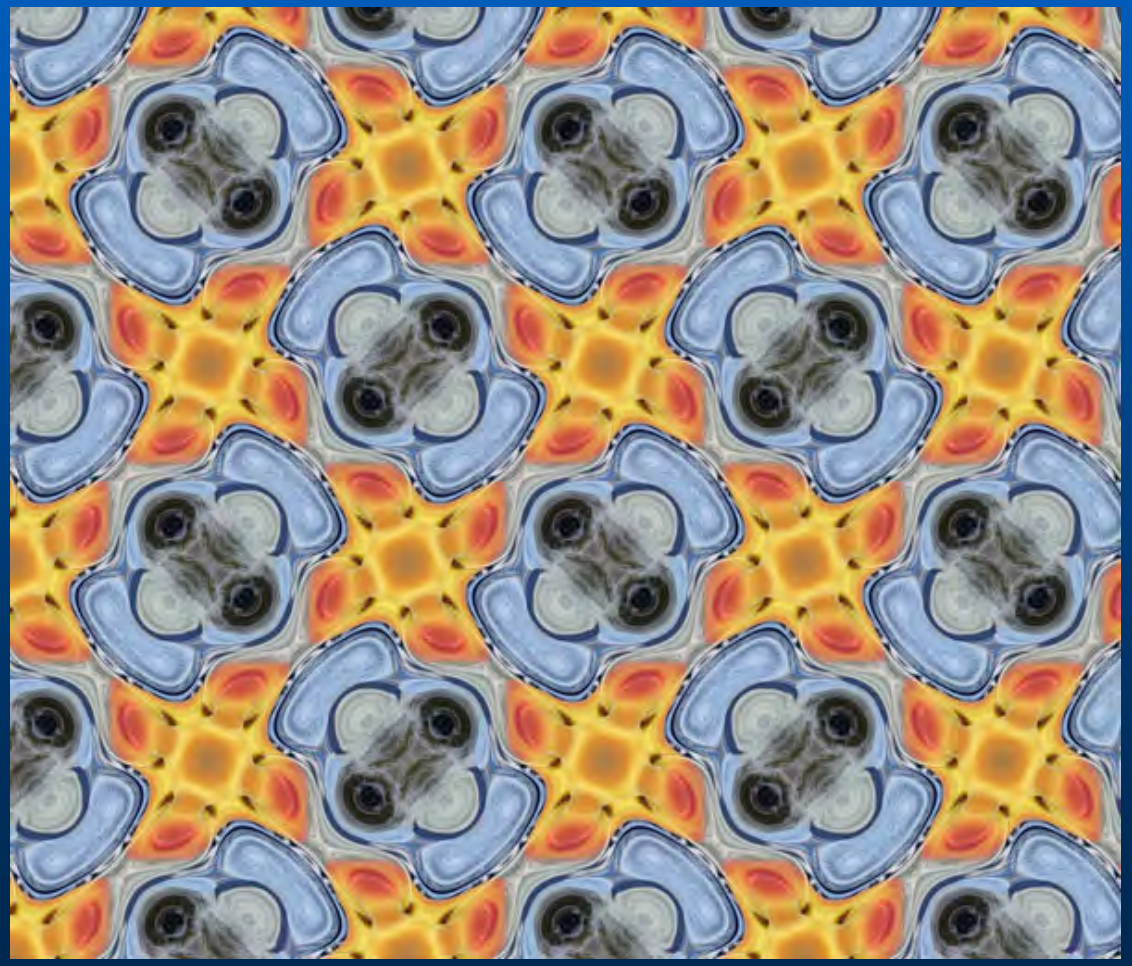
This group is called **p4g** by the International Union of Crystallographers

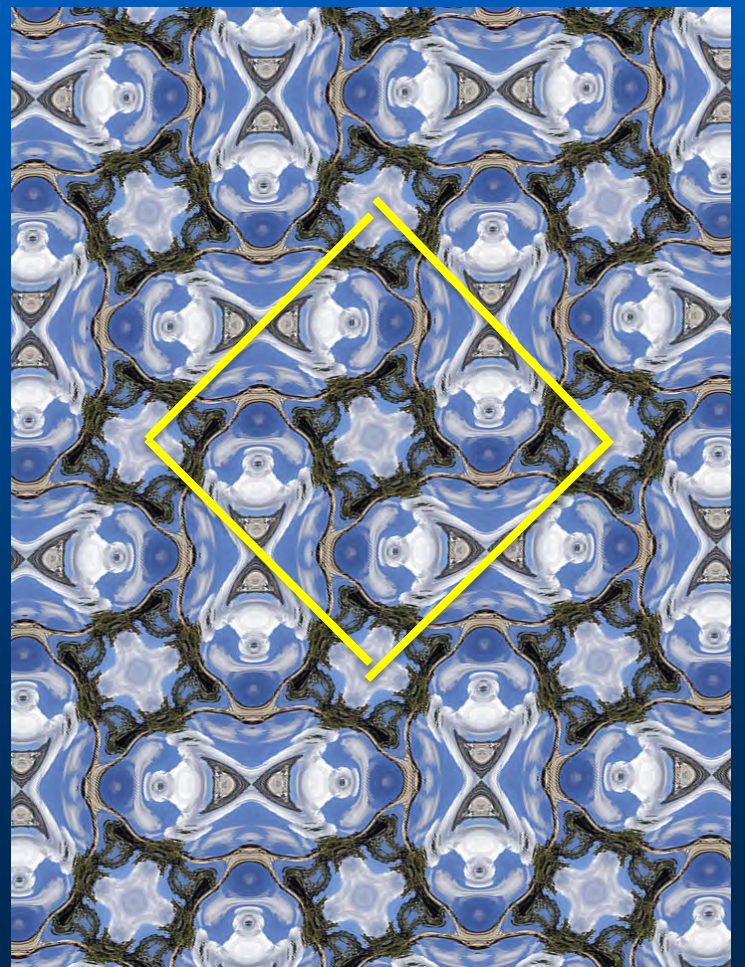


p4g can be generated by one mirror, one rotation



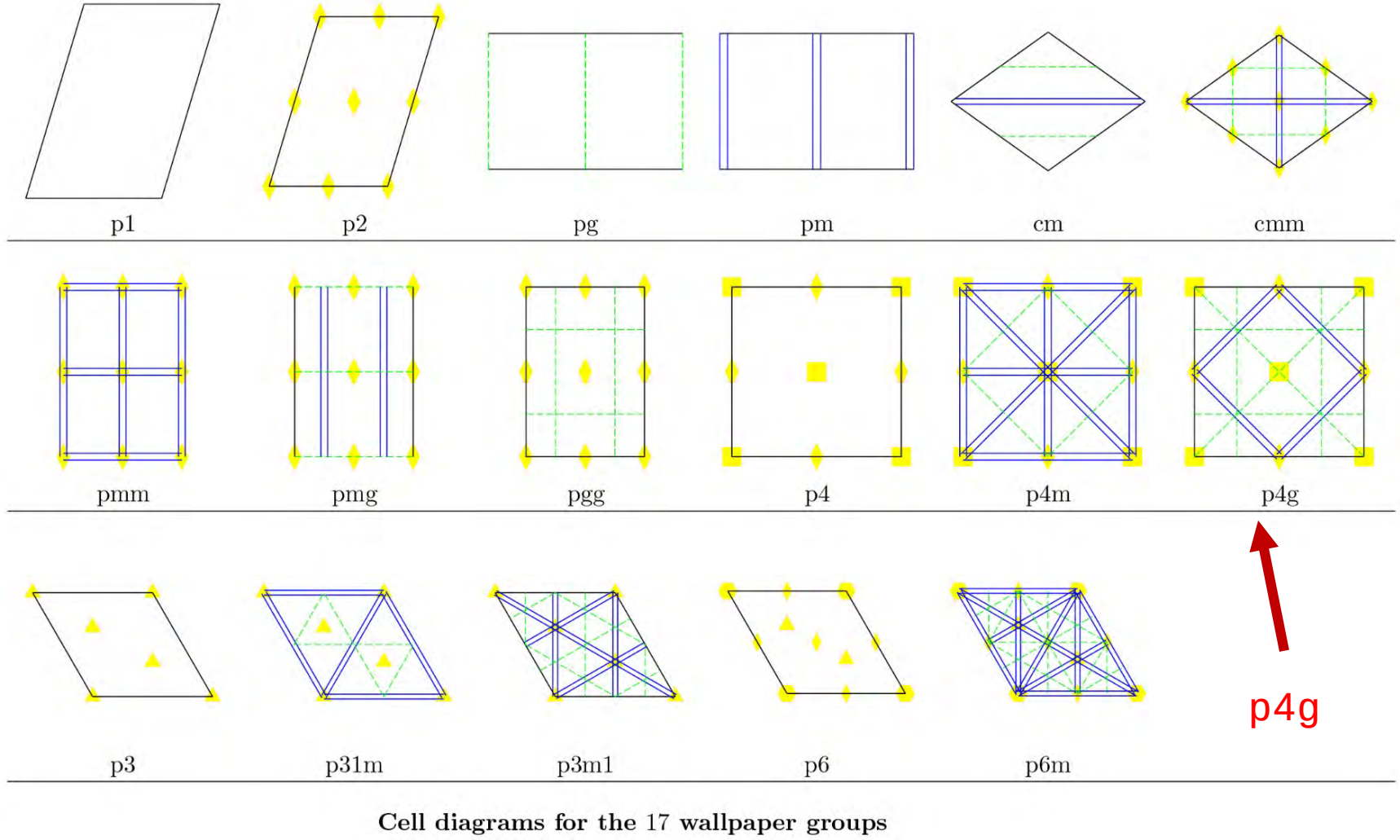
Two patterns have the same “type” if their symmetry groups are the same





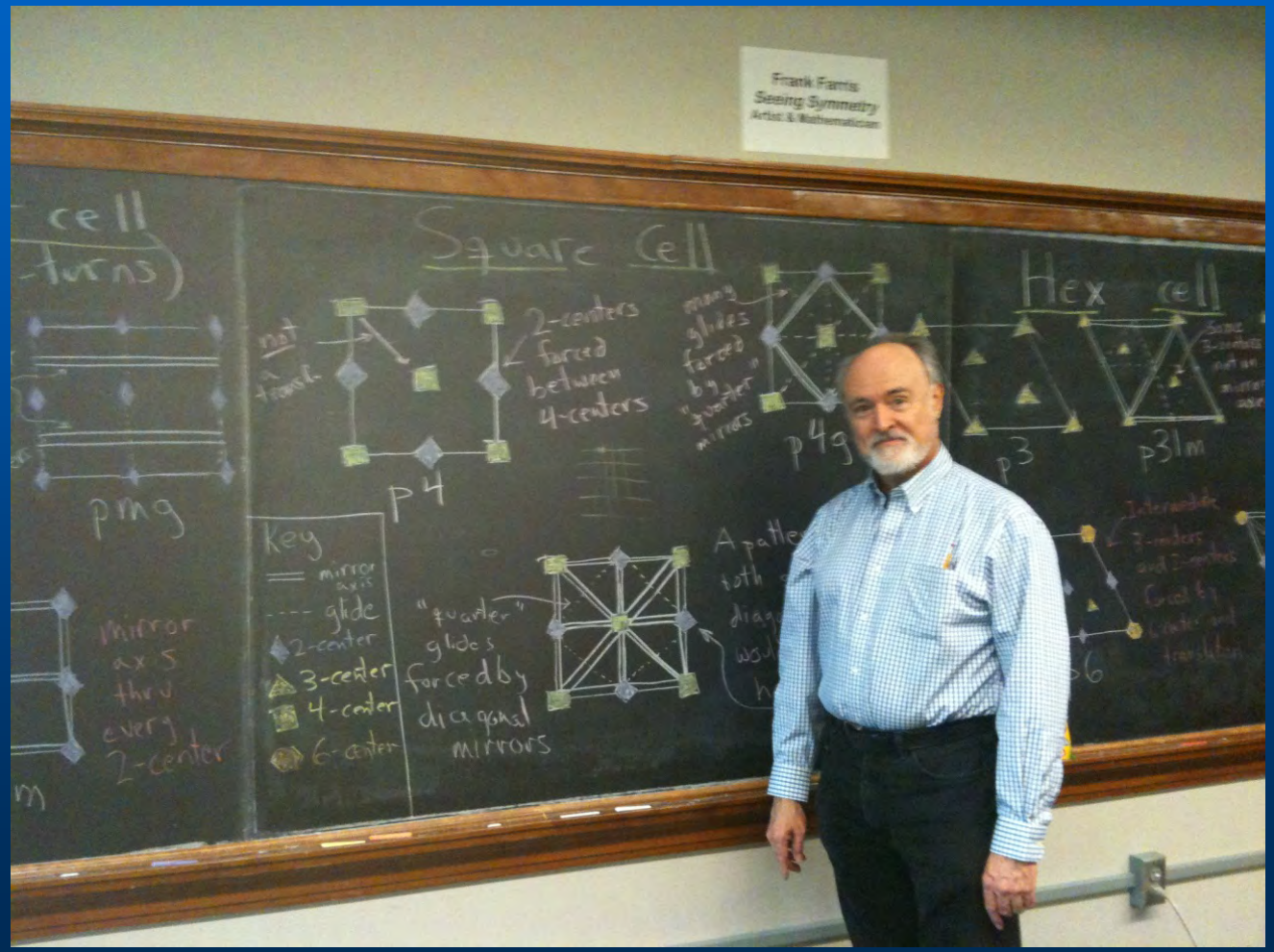
Wallpaper surprise:

Exactly 17 types



Cell diagrams for the 17 wallpaper groups

Chalk Slam at Carleton

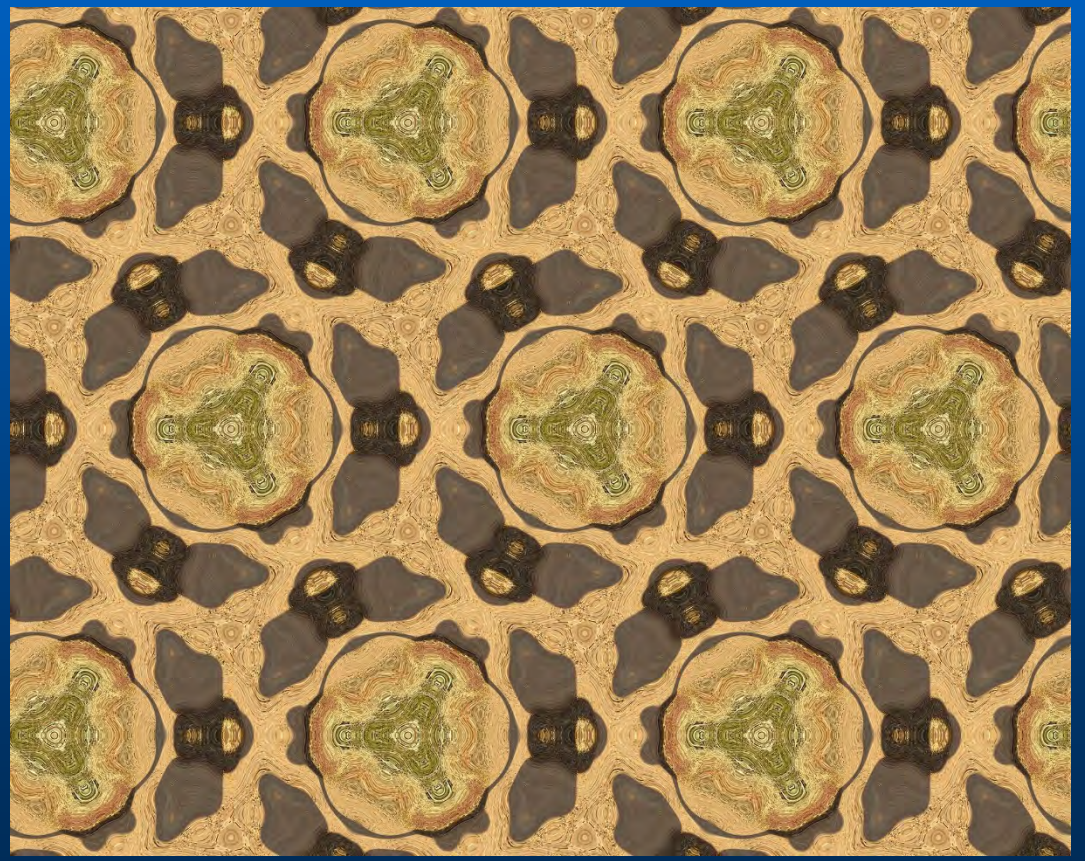


See them everywhere!

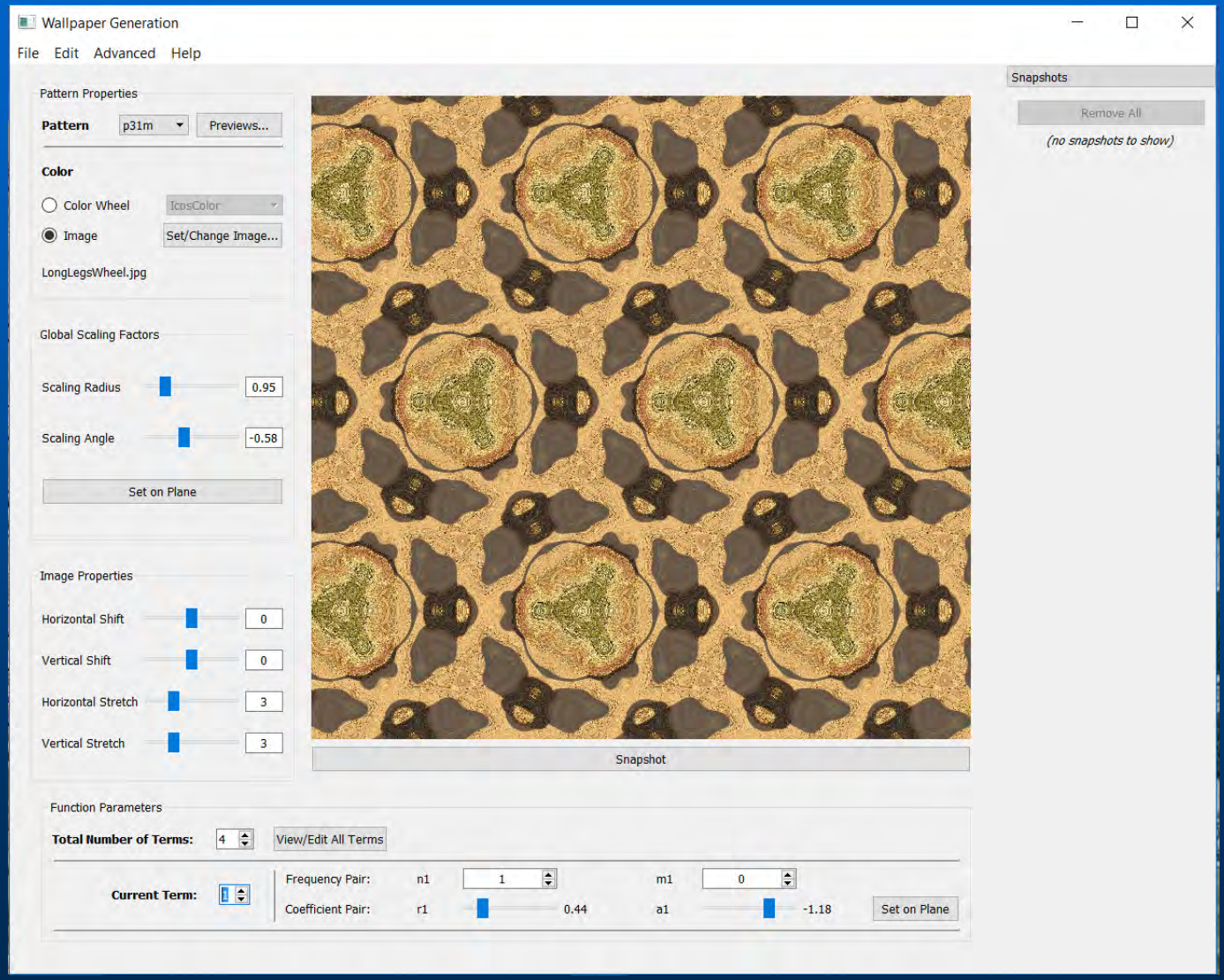
pmg
pattern from
The Shining

Construct wallpaper functions

$$f(g(z)) = f(z) \text{ for } g \in G$$



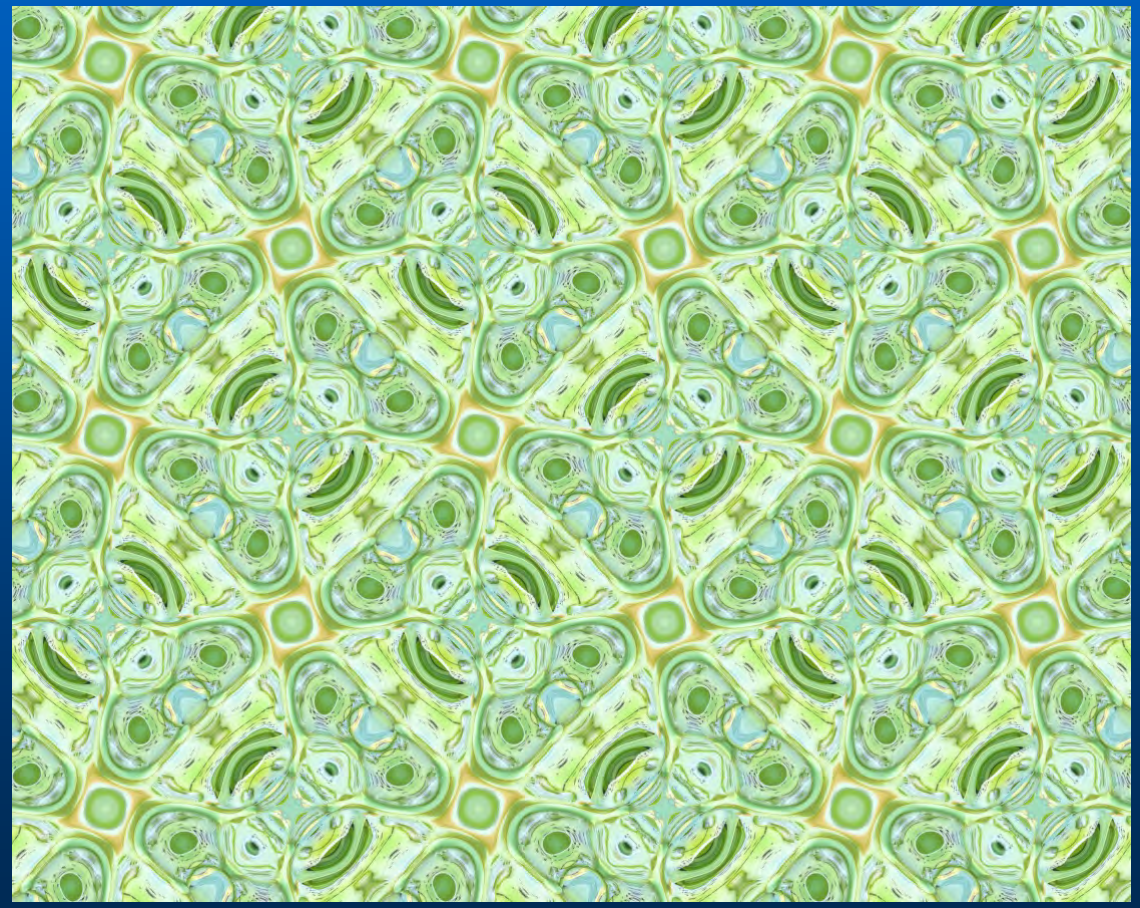
SymmetryWorks screenshot



Written by students at Bowdoin College, based on software by SCU students. Handout available

A few favorites: Minimalist

Recognize p4g?



A few favorites: Fanciful

Machine learning project:
Find patterns that
humans will like

Color-reversing (or 2-color) symmetry

$$
\begin{aligned}
f(g(z)) &= f(z) \text{ for some} \\
f(h(z)) &= -f(z) \text{ for others}
\end{aligned}
$$

Requires a color-reversing wheel

And the right waveforms

A break for algebra: Suppose

$$f(h(z)) = -f(z) \text{ for some transformation } h$$

$$G_c = \{g | f(g(z)) = \pm f(z)\}$$

$$G = \{g | f(g(z)) = f(z)\}$$

Color gp
Sym. gp

Homomorphism

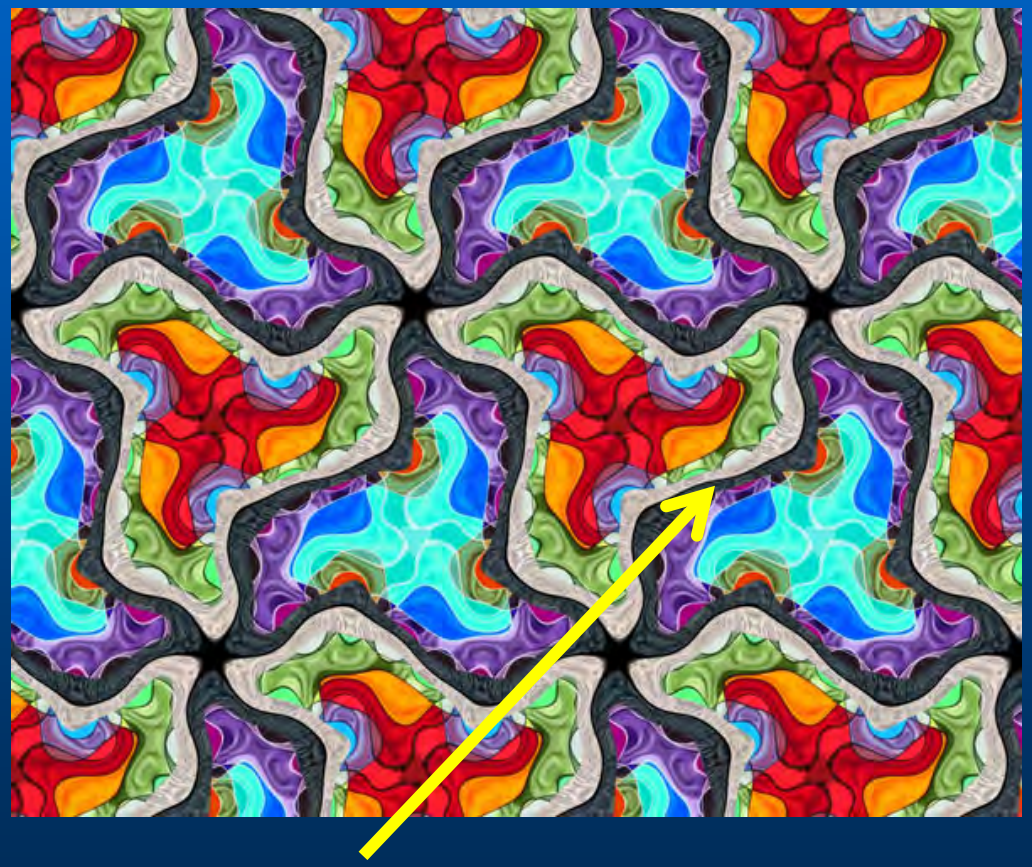
$$\phi : G_c \to \{1, -1\}$$

G is normal in G_c of index 2

$$f(g(z)) = \phi(g) f(z) \text{ for all } g \in G_c$$

How many ways?

17 symmetry types
46 color-reversing types

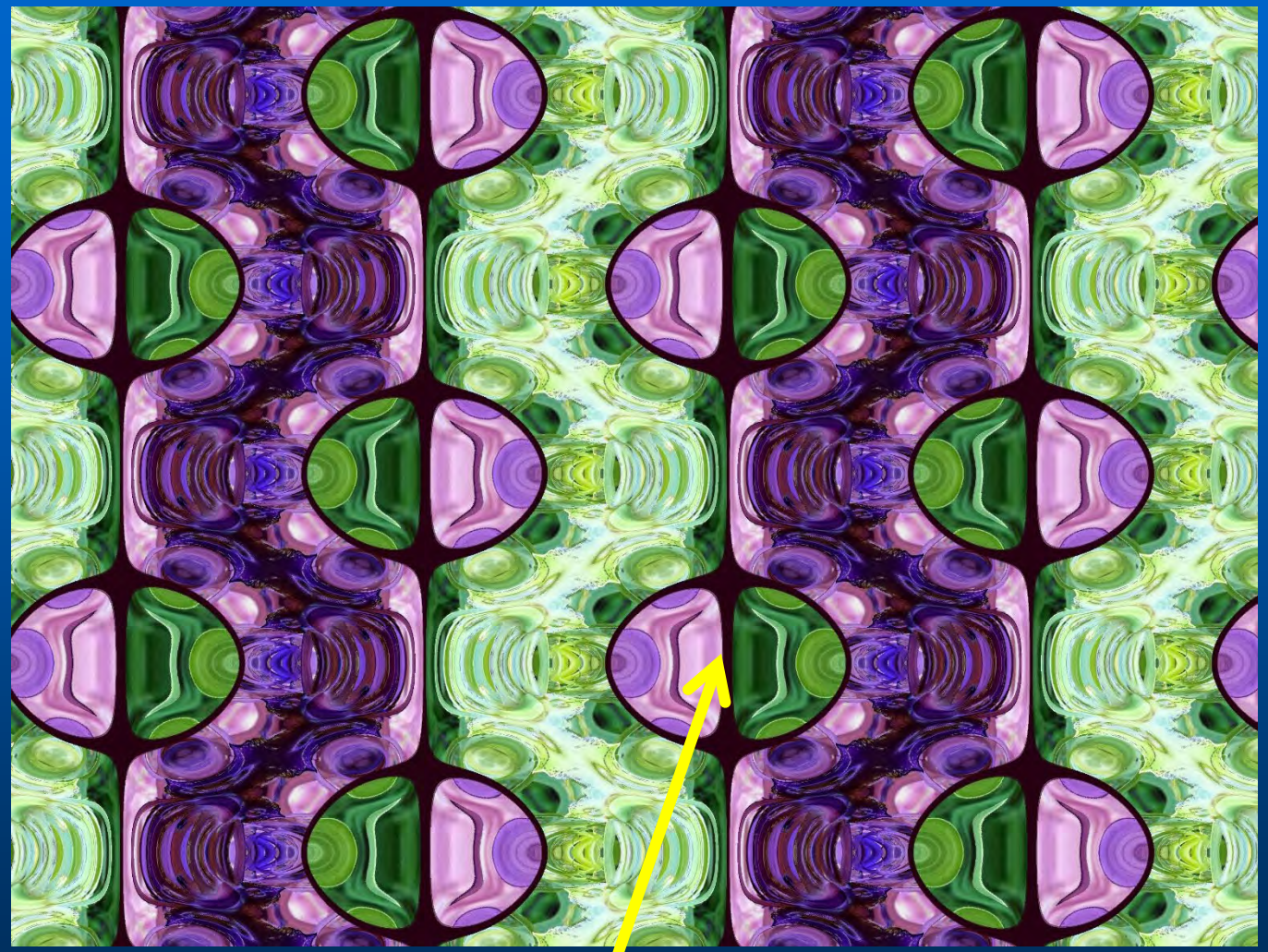


Color-reversing half turn

Actual symmetry group: p3

Color-reversing or –preserving symmetries: p6

type: p6/p3



Actual symmetry group: pmg

Color-reversing or –preserving symmetries: cmm

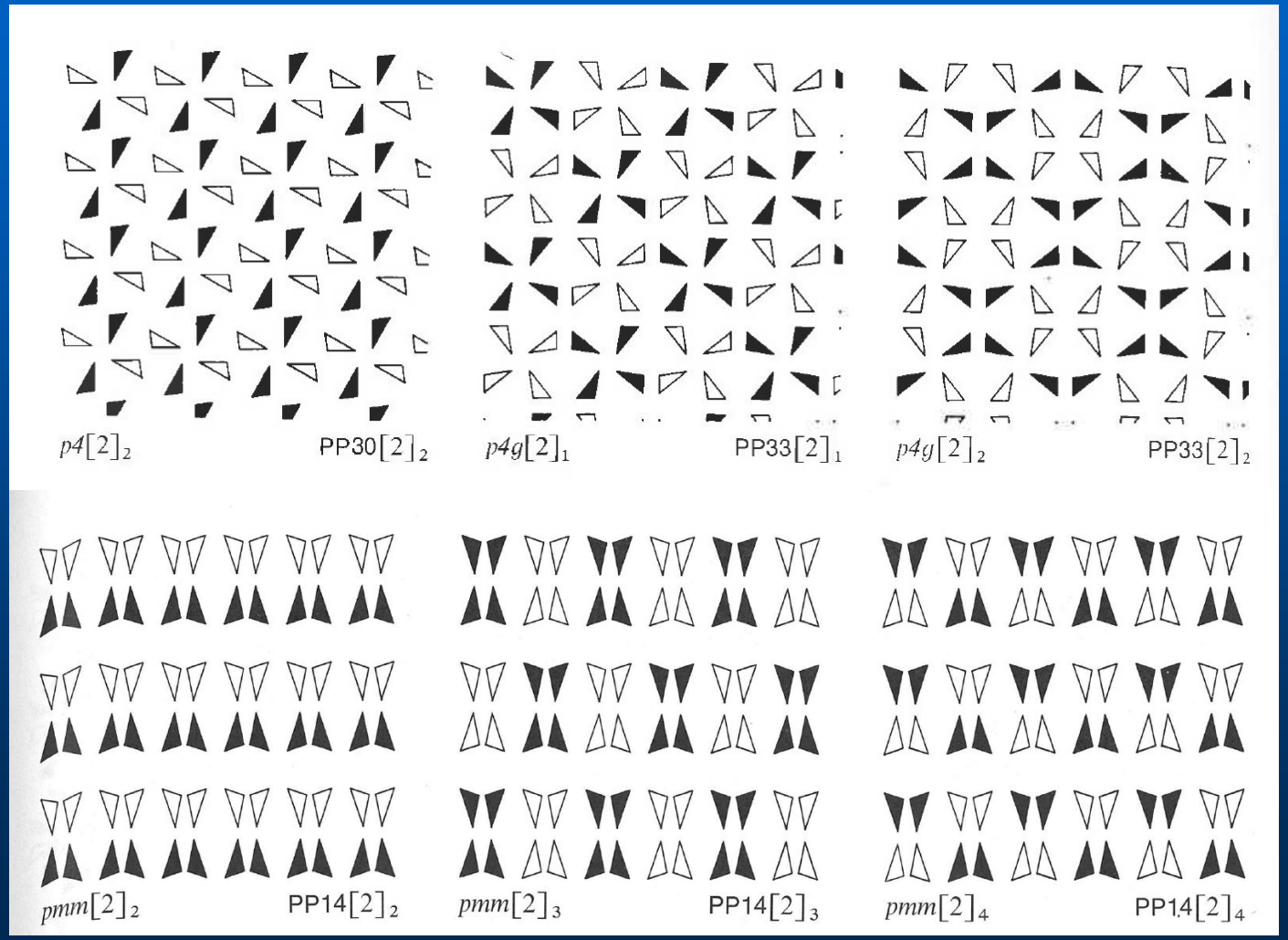
type: cmm/pmg

Color black counts as 0

Color-reversing half turn

THE GEOMETRICAL BASIS OF PATTERN DESIGN. Part IV: Counterchange Symmetry in Plane Patterns, by H. J. Woods

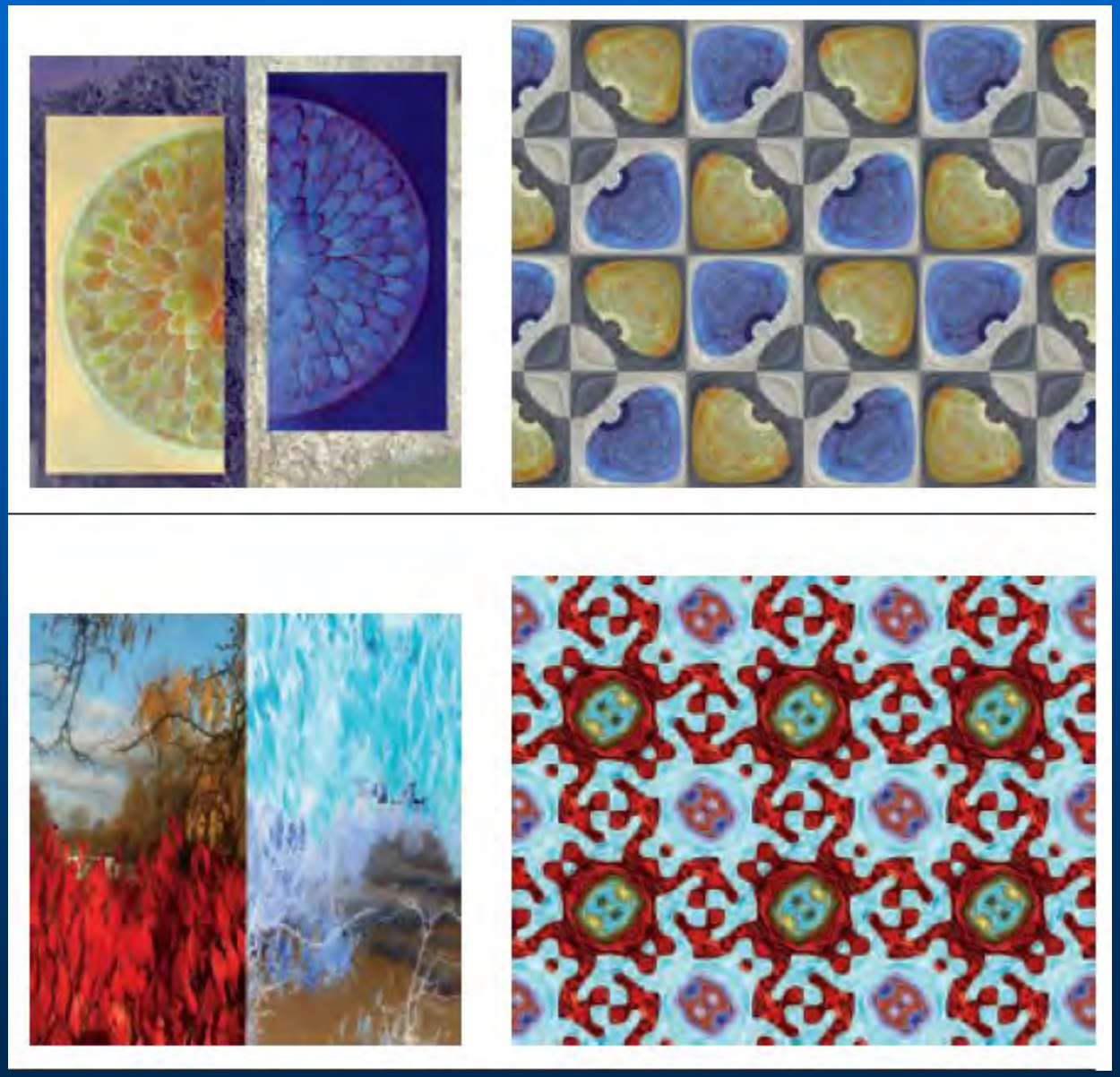
Journal of Textiles (Manchester) 1936



Figures from Grünbaum and Shephard, *Tilings and Patterns*

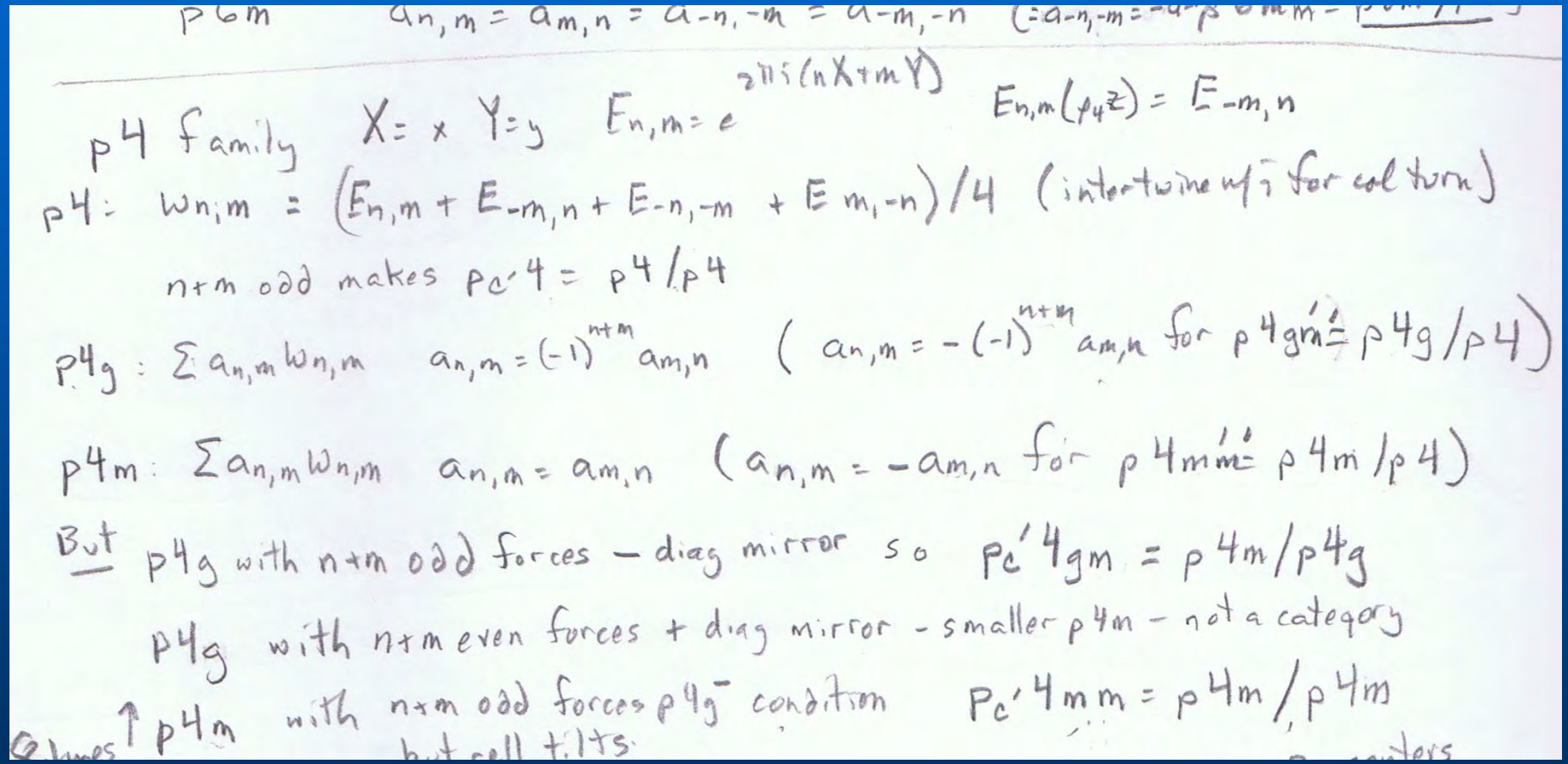
Ad hoc notations like p4g[2]_2

Figures from appendix of *Creating Symmetry*



Types
p4m/cmm and
p4g/cmm

Recipes for 63 types



These recipes are not encoded in current software

Three-color (color-turning) symmetry

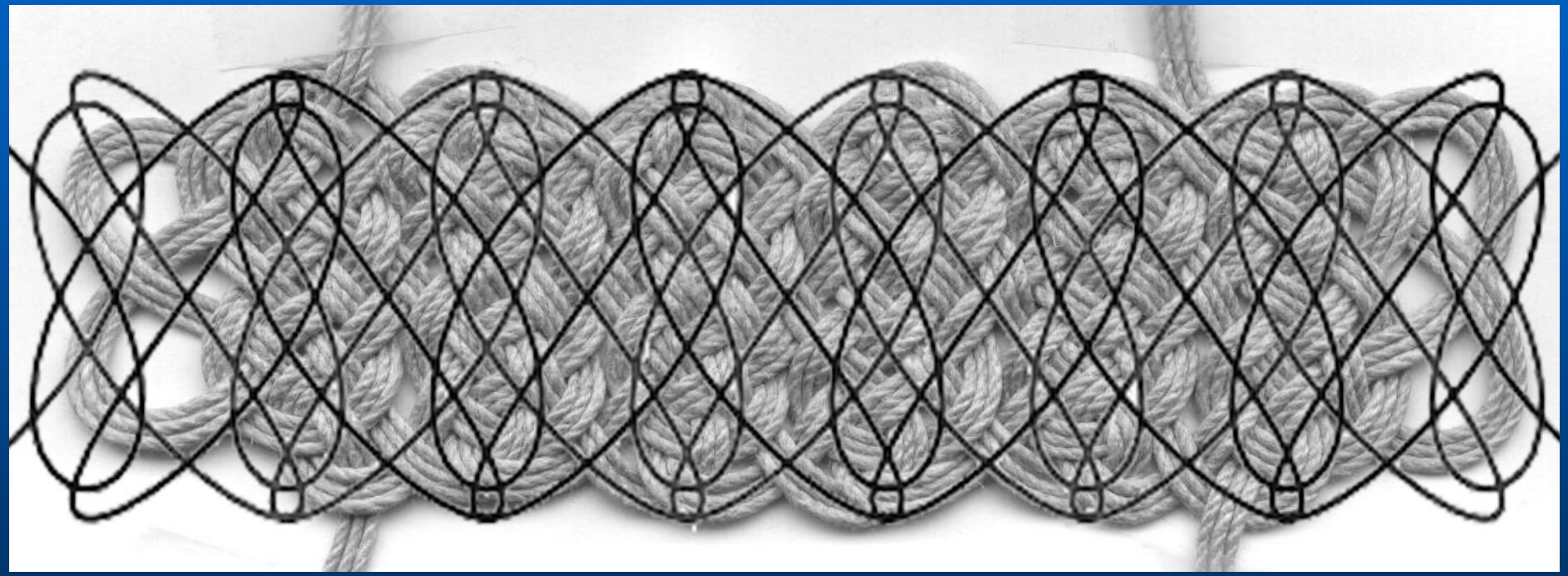
$$f(g(z)) = \phi(g) f(z) \text{ for all } g \in G_c$$

$$f(\omega z) = \omega f(z)$$

Three-color (color-turning) symmetry

Diagonal 1/3 translations are color-turning

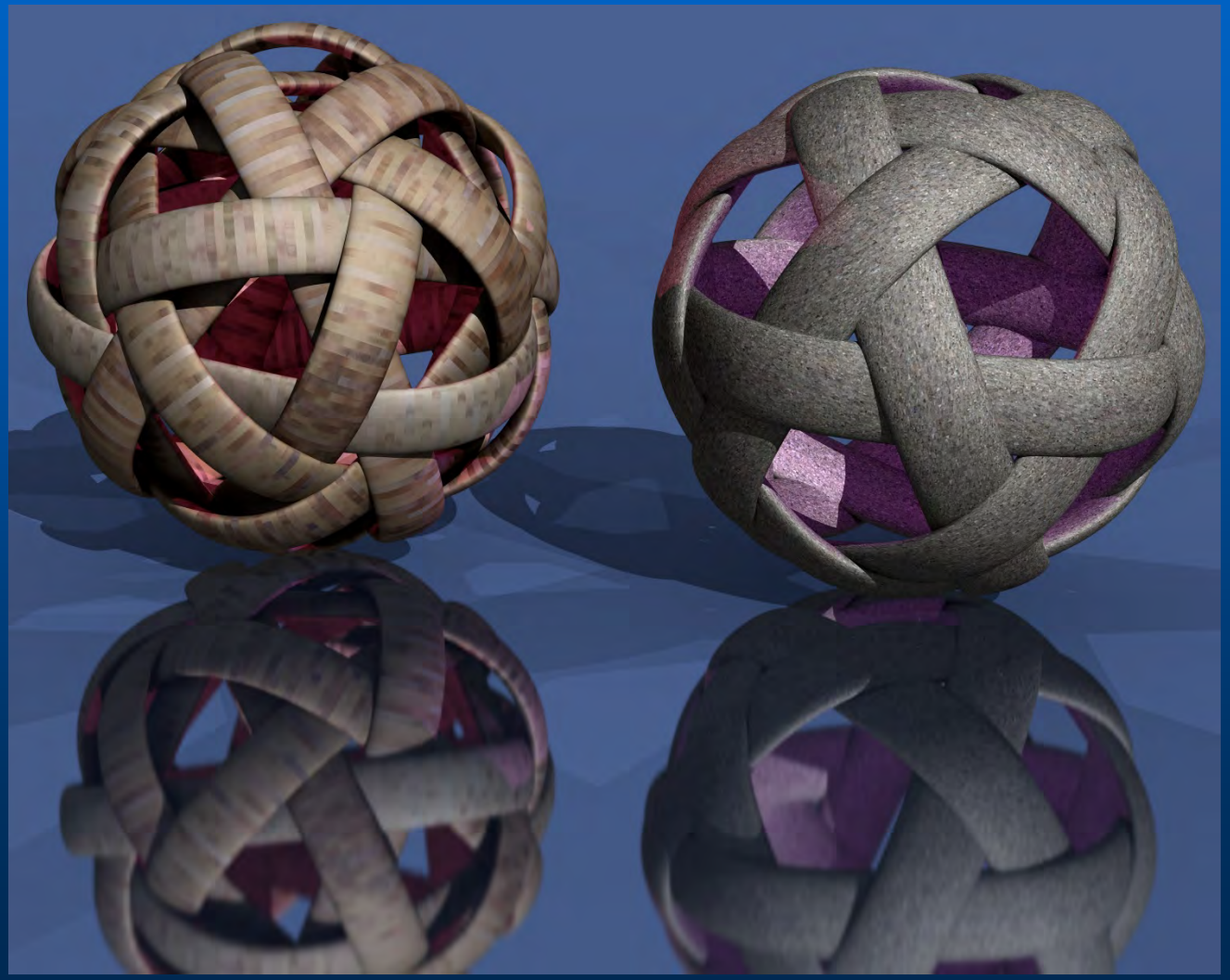
Connect to the non-digital world?



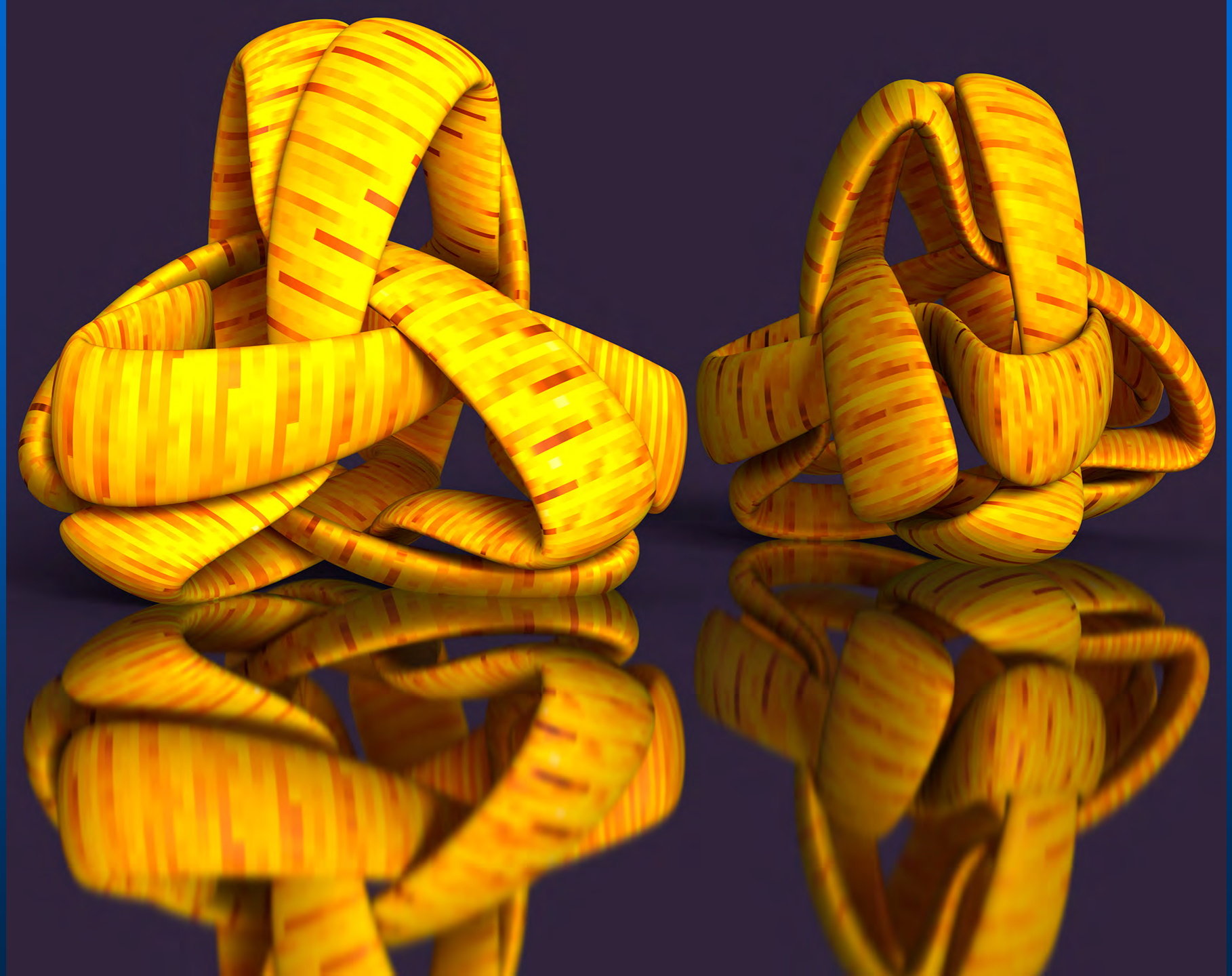
Woven Rope Friezes, with Hans Kristian Rossing in Mathematics Magazine, 1999

Physical object not invariant under mirror symmetry, but rather under half-turn symmetry in space

Connect to the non-digital world?



"Woven" polyhedral shapes



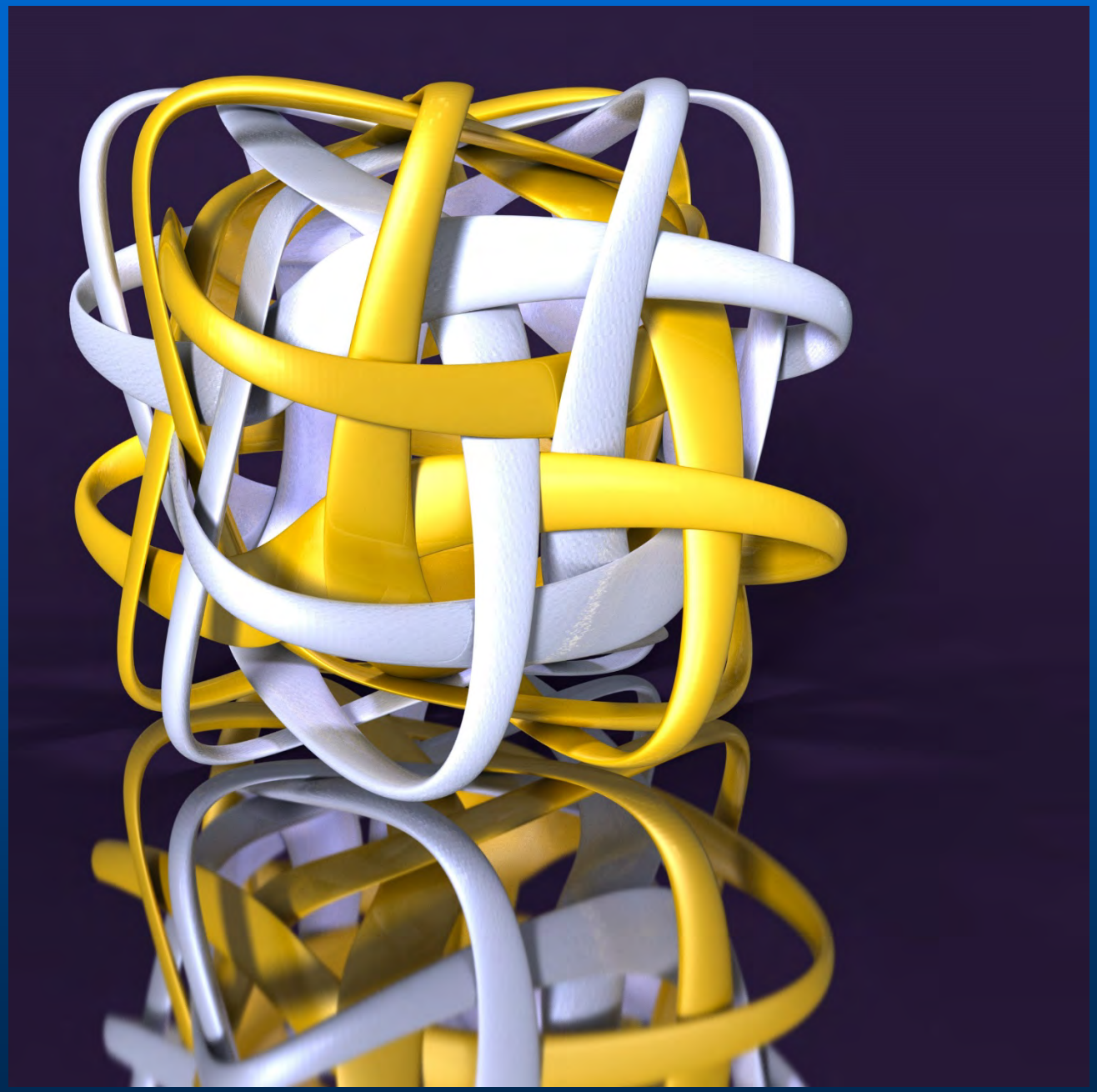
Color gp

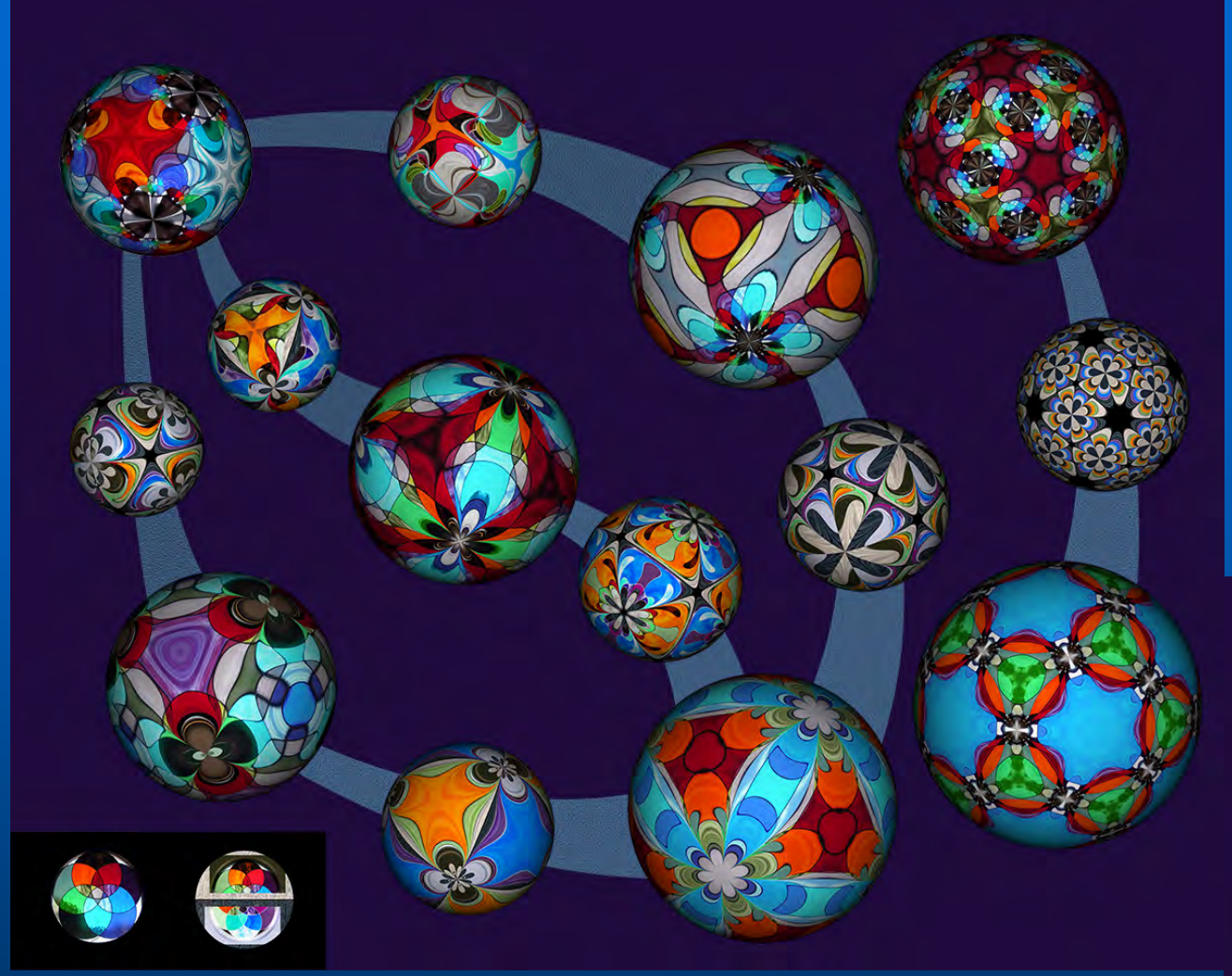
O

Sym. gp

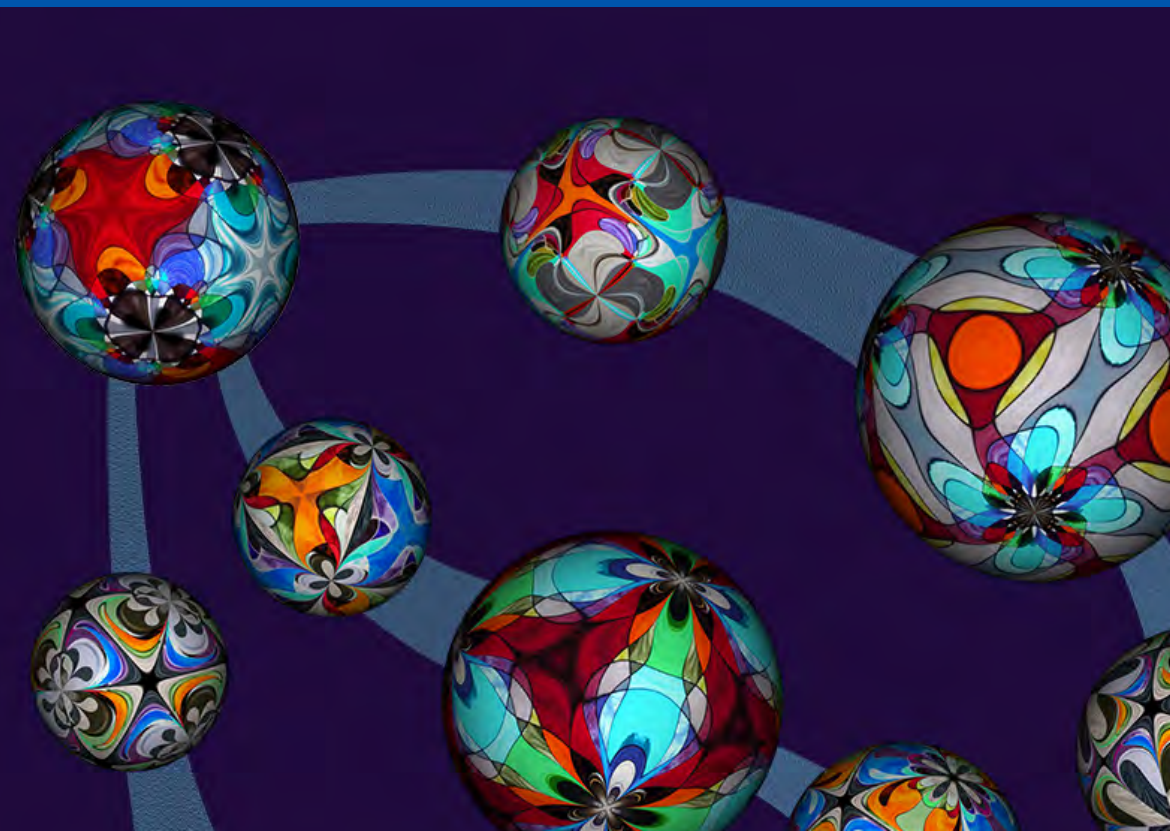
T

24 Bands





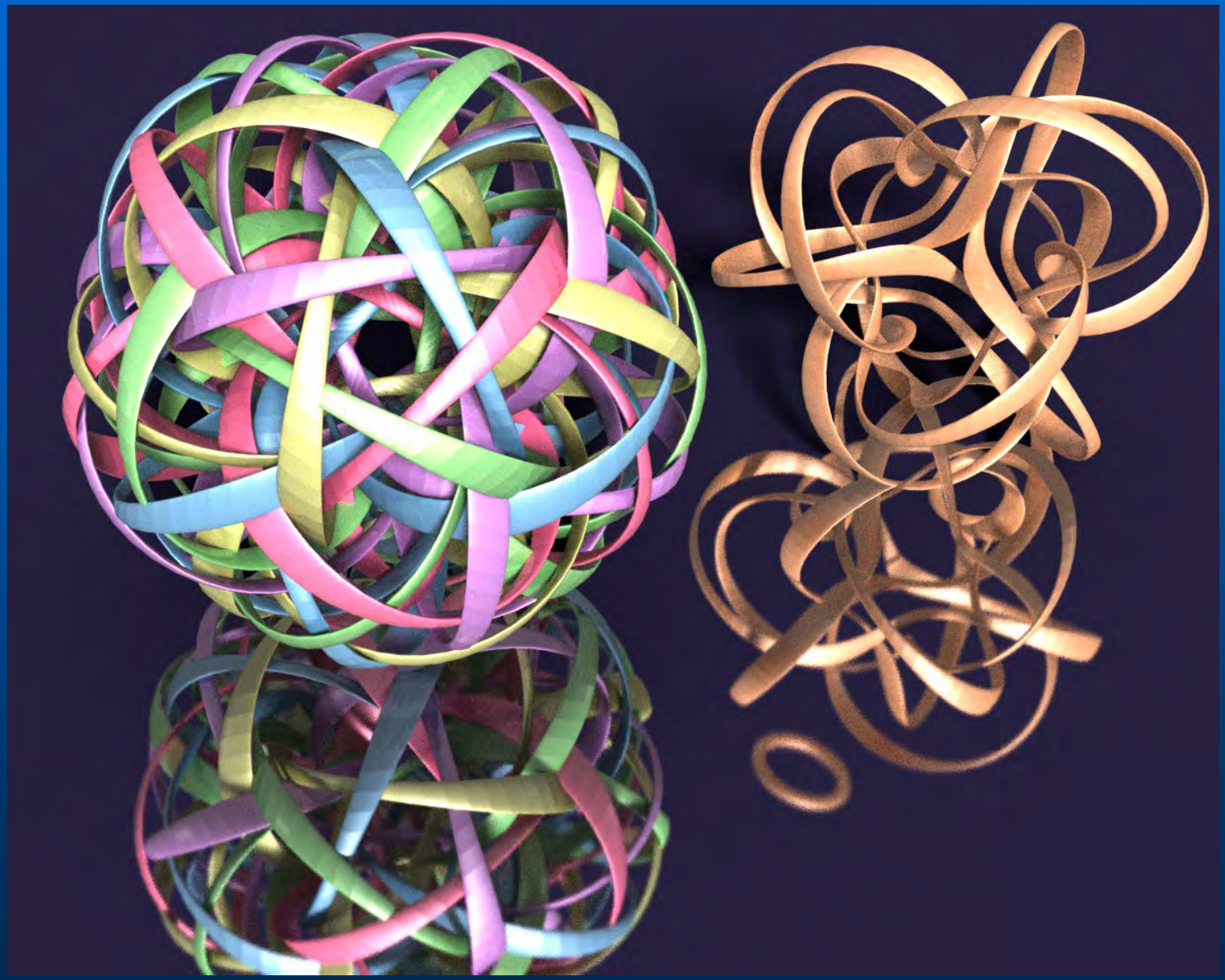
Aside: Polyhedral Sampler from 2015

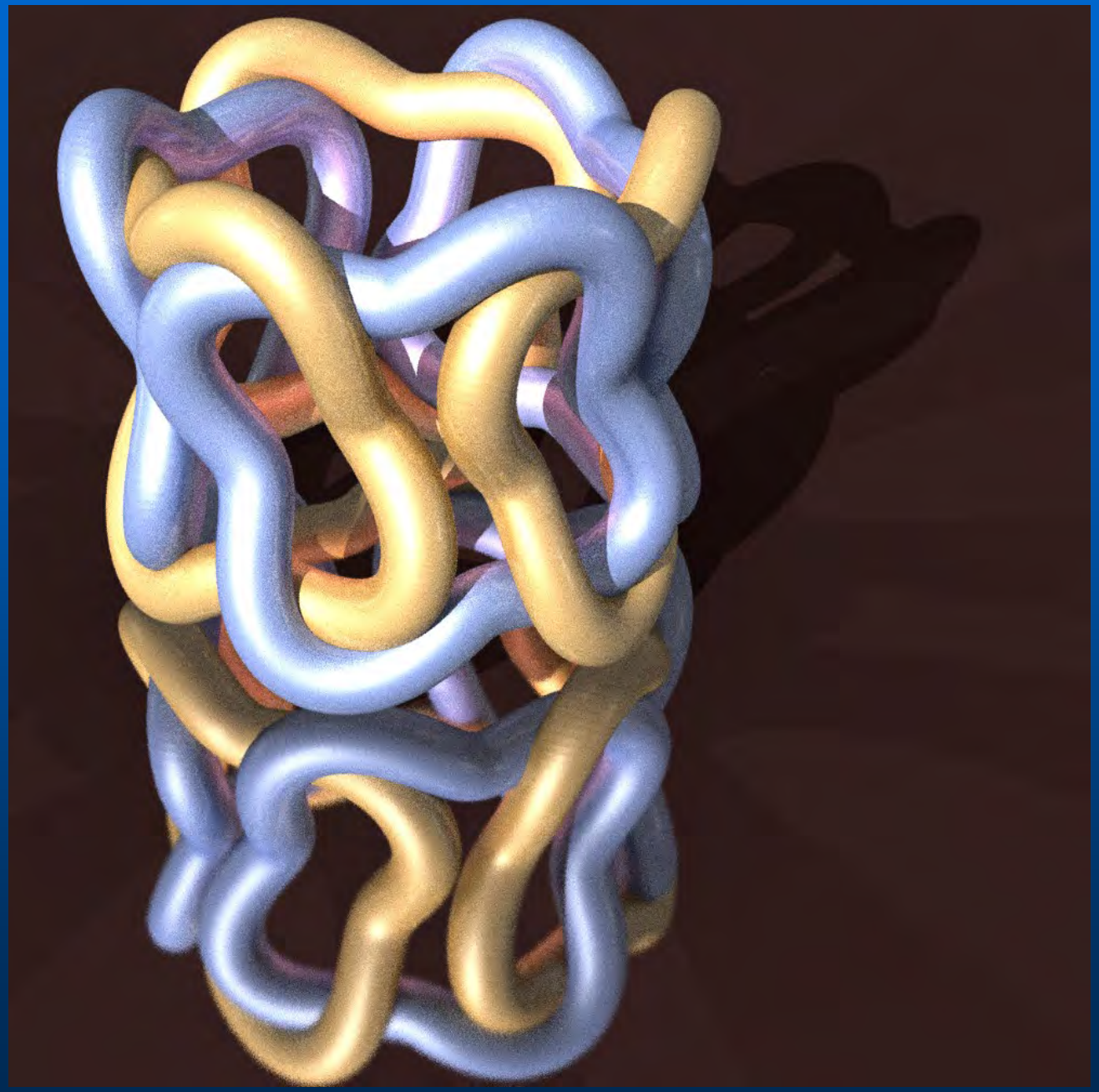


Icos from five tetrahedra

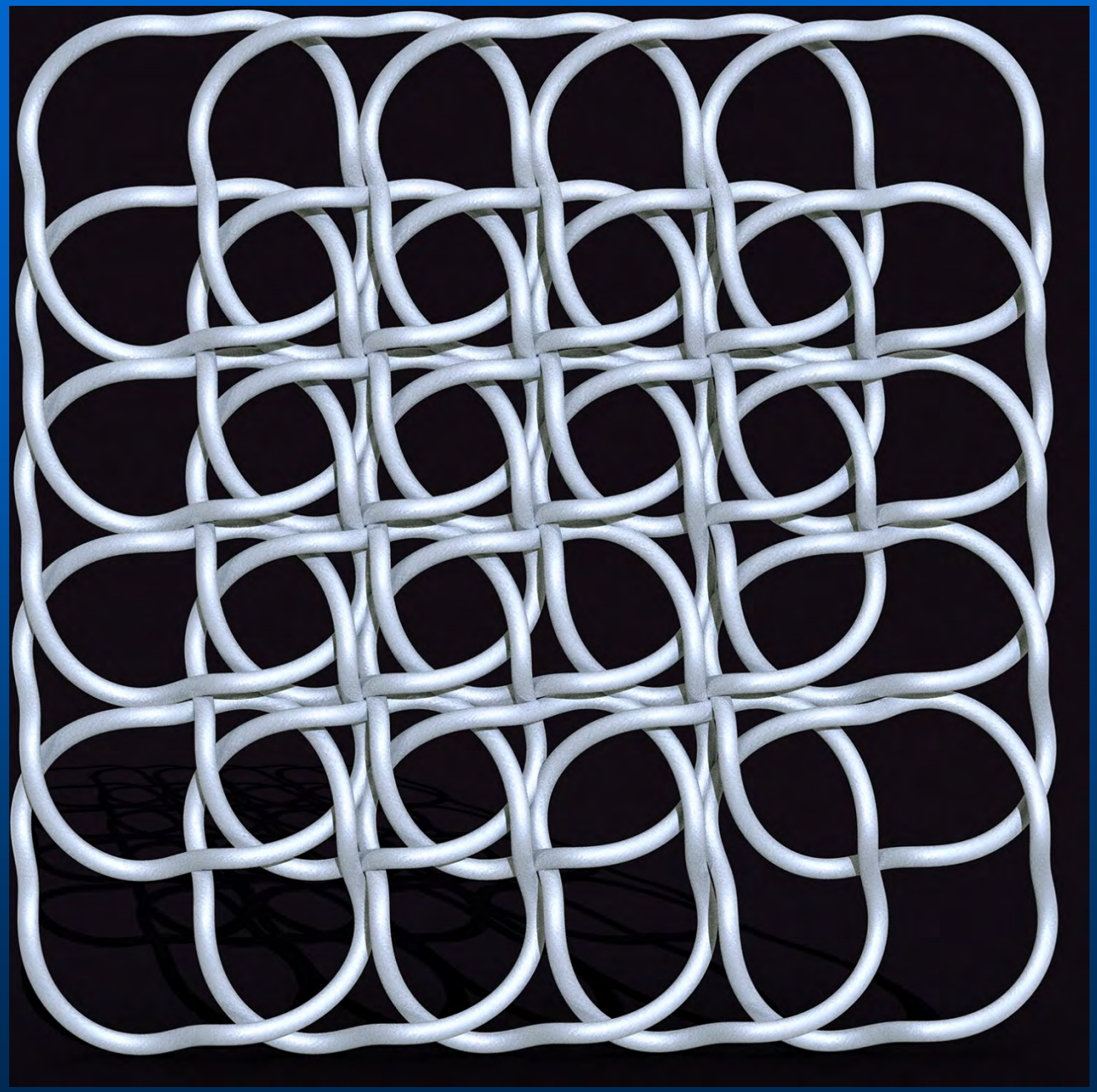
60 bands.

Does *not* have 5-color symmetry





D6/D3



Question: Which frieze and wallpaper patterns can be woven in chain mail?

Color patterns?